

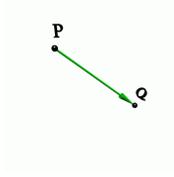
DISTANCES

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DISTANCE POINT-POINT (3D). If P and Q are two points, then

$$d(P, Q) = |\vec{PQ}|$$

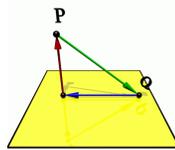
is the distance between P and Q .



DISTANCE POINT-PLANE (3D). If P is a point in space and $\Sigma : \vec{n} \cdot \vec{x} = d$ is a plane containing a point Q , then

$$d(P, \Sigma) = |(\vec{PQ} \cdot \vec{n}) / |\vec{n}|$$

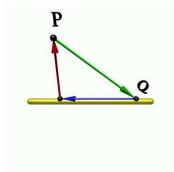
is the distance between P and the plane.



DISTANCE POINT-LINE (3D). If P is a point in space and L is the line $\vec{r}(t) = Q + t\vec{u}$, then

$$d(P, L) = |(\vec{PQ} \times \vec{u}) / |\vec{u}|$$

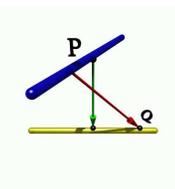
is the distance between P and the line L .



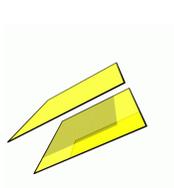
DISTANCE LINE-LINE (3D). L is the line $\vec{r}(t) = Q + t\vec{u}$ and M is the line $\vec{s}(t) = P + t\vec{v}$, then

$$d(L, M) = |(\vec{PQ} \cdot (\vec{u} \times \vec{v})) / |\vec{u} \times \vec{v}|$$

is the distance between the two lines L and M .



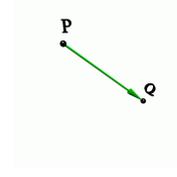
DISTANCE PLANE-PLANE (3D). If $\vec{n} \cdot \vec{x} = d$ and $\vec{n} \cdot \vec{x} = e$ are two parallel planes, then their distance is $(e - d) / |\vec{n}|$. Nonparallel planes have distance 0.



EXAMPLES

DISTANCE POINT-POINT (3D). $P = (-5, 2, 4)$ and $Q = (-2, 2, 0)$ are two points, then

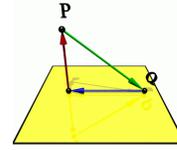
$$d(P, Q) = |\vec{PQ}| = \sqrt{(-5+2)^2 + (2-2)^2 + (0-4)^2} = 5$$



DISTANCE POINT-PLANE (3D). $P = (7, 1, 4)$ is a point and $\Sigma : 2x + 4y + 5z = 9$ is a plane which contains the point $Q = (0, 1, 1)$. Then

$$d(P, \Sigma) = |(-7, 0, -3) \cdot (2, 4, 5) / |\sqrt{45}| = 29 / \sqrt{45}$$

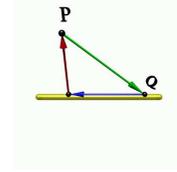
is the distance between P and Σ .



DISTANCE POINT-LINE (3D). $P = (2, 3, 1)$ is a point in space and L is the line $\vec{r}(t) = (1, 1, 2) + t(5, 0, 1)$. Then

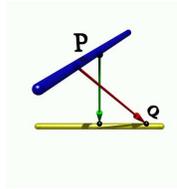
$$d(P, L) = |(-1, -2, 1) \times (5, 0, 1) / \sqrt{26}| = |(-2, 6, 10) / \sqrt{26}| = \sqrt{140} / \sqrt{26}$$

is the distance between P and L .



DISTANCE LINE-LINE (3D). L is the line $\vec{r}(t) = (2, 1, 4) + t(-1, 1, 0)$ and M is the line $\vec{s}(t) = (-1, 0, 2) + t(5, 1, 2)$. The cross product of $(-1, 1, 0)$ and $(5, 1, 2)$ is $(2, 2, -6)$. The distance between these two lines is

$$d(L, M) = |(3, 1, 2) \cdot (2, 2, -6) / \sqrt{44}| = 4 / \sqrt{44}$$



DISTANCE PLANE-PLANE (3D). $5x + 4y + 3z = 8$ and $5x + 4y + 3z = 1$ are two parallel planes. Their distance is $7 / \sqrt{50}$.

