4/18/2006 SECOND HOURLY SECOND PRACTICE Math 21a, Spring 2006

Name:

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (30 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) [T] [F] $f(x,y)$ and $g(x,y) = f(x^2, y^2)$ have the same critical points.

2) [T] [F] If a function $f(x,y) = ax + by$ has a critical point, then $f(x,y) = 0$ for all (x,y).

3) [T] [F] Given 2 arbitrary points in the plane, there is a function $f(x,y)$ which has these points as critical points and no other critical points.

4) [T] [F] If (x_0, y_0) is the maximum of $f(x,y)$ on the disc $x^2+y^2 \leq 1$ then $x_0^2+y_0^2 < 1$.

5) [T] [F] There are no functions $f(x,y)$ for which every point on the unit circle is a critical point.

6) [T] [F] An absolute maximum (x_0, y_0) of $f(x,y)$ is also an absolute maximum of $f(x,y)$ constrained to a curve $g(x,y) = c$ that goes through the point (x_0, y_0).

7) [T] [F] If $f(x,y)$ has two local maxima on the plane, then f must have a local minimum on the plane.

8) [T] [F] There exists a function $f(x,y)$ of two variables which has no critical points at all.

9) [T] [F] If $f_x(x,y) = f_y(x,y) = 0$ for all (x,y) then $f(x,y) = 0$ for all (x,y) .

10) [T] [F] $(0,0)$ is a local maximum of the function $f(x,y) = x^2 - y^2 + x^4 + y^4$.

11) [T] [F] If $f(x,y)$ has a local maximum at the point $(0,0)$ with discriminant $D > 0$ then $g(x,y) = f(x,y) - x^4 + y^3$ has a local maximum at the point $(0,0)$ too.

12) [T] [F] Every critical point (x,y) of a function $f(x,y)$ for which the discriminant D is not zero is either a local maximum or a local minimum.

13) [T] [F] If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) < 0$ then $(0,0)$ can not be a local minimum.

14) [T] [F] In the second derivative test, one can replace the condition $D > 0, f_{xx} > 0$ with $D > 0, f_{yy} > 0$ to check whether a point is a local minimum.

15) [T] [F] The function $f(x,y) = (x^4 - y^4)$ has neither a local maximum nor a local minimum at $(0,0)$.

16) [T] [F] It is possible to find a function of two variables which has no maximum and no minimum.

17) [T] [F] $\int_0^2 \int_0^2 (x^2+y^2)\cos(x^3+y^3)\, dxdy \leq 32$.

18) [T] [F] $\int_0^2 \int_0^{x^2} f(x,y)\, dydx = \int_0^4 \int_{\sqrt{y}}^2 f(x,y)\, dxdy$.

19) [T] [F] The area of a polar region $0 \leq r \leq r(\theta)$ is $\int_0^{2\pi} r(\theta)^2/2\, d\theta$.

20) [T] [F] If R is the unit disc in the xy-plane, then $\int\int_R -\sqrt{1-x^2-y^2}\ dxdy = -2\pi/3$.

Problem 2) (10 points)

Match the integrals with those obtained by changing the order of integration. No justifications are needed.

Enter I,II,III,IV or V here.	Integral
	$\int_0^1 \int_{1-y}^1 f(x,y)\, dxdy$
	$\int_0^1 \int_y^1 f(x,y)\, dxdy$
	$\int_0^1 \int_0^{1-y} f(x,y)\, dxdy$
	$\int_0^1 \int_0^y f(x,y)\, dxdy$

I) $\int_0^1 \int_0^x f(x,y)\, dydx$

II) $\int_0^1 \int_0^{1-x} f(x,y)\, dydx$

III) $\int_0^1 \int_x^1 f(x,y)\, dydx$

IV) $\int_0^1 \int_0^{x-1} f(x,y)\, dydx$

V) $\int_0^1 \int_{1-x}^1 f(x,y)\, dydx$

Problem 3) (10 points)

Which point on the surface $g(x,y,z) = \frac{1}{x} + \frac{1}{y} + \frac{8}{z} = 1$ is closest to the origin?

Problem 4) (10 points)

Find all extrema of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$ on the plane and characterize them. Do you find a absolute maximum or absolute minimum among them?

Problem 5) (10 points)

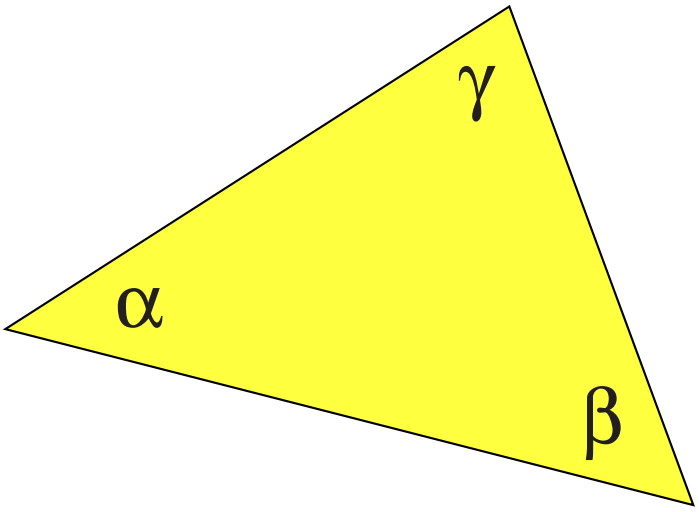
Find all the critical points of $f(x,y) = \frac{x^5}{5} - \frac{x^2}{2} + \frac{y^3}{3} - y$ and indicate whether they are local maxima, local minima or saddle points.

Problem 6) (10 points)

What is the shape of the triangle with angles α, β, γ for which

$$f(\alpha,\beta,\gamma) = \log\left(\sin(\alpha)\sin(\beta)\sin(\gamma)\right)$$

is maximal?



Problem 7) (10 points)

Evaluate the integral

$$\int_{\pi/4}^{3\pi/4} \int_{1/\sin(\theta)}^{2\sin(\theta)} r \; dr d\theta$$

Hint: There is not much to compute if you know how the region looks like.

Problem 8) (10 points)

Evaluate the integral

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin(y)}{y} \, dydx \, .$$

Problem 9) (10 points)

Find the surface area of the surface parametrized by

$$\vec{r}(u, v) = \langle u, v, 2u - v \rangle$$

with $1 \leq u \leq 2$, $-1 \leq v \leq 1$.

Problem 10) (10 points)

Integrate the function $f(x, y, z) = x^2 + y^2$ over the solid bound by the planes $z = 1, z = -1$ and the hyperboloid $x^2 + y^2 - z^2 = 1$.