Name:

MWF 10 Samik Basu
MWF 10 Joachim Krieger
MWF 11 Matt Leingang
MWF 11 Veronique Godin
TTH 10 Oliver Knill
TTH 115 Thomas Lam

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F $(1,1)$ is a local maximum of the function $f(x,y) = x^2y - x + \cos(y)$.

2) T F If f is a smooth function of two variables, then the number of critical points of f inside the unit disc is finite.

3) T F If $(1,1)$ is a critical point for the function $f(x,y)$ then $(1,1)$ is also a critical point for the function $g(x,y) = f(x^2,y^2)$.

4) T F There is no function $f(x,y,z)$ of three variables, for which every point on the unit sphere is a critical point.

5) T F If (x_0,y_0) is a maximum of $f(x,y)$ under the constraint $g(x,y) = g(x_0,y_0)$, then (x_0,y_0) is a maximum of $g(x,y)$ under the constraint $f(x,y) = f(x_0,y_0)$.

6) T F If $\vec{u}$ is a unit vector tangent at (x,y,z) to the level surface of $f(x,y,z)$ then $D_u f(x,y,z) = 0$.

7) T F The vector $\vec{r}_u - \vec{r}_v$ is tangent to the surface parameterized by $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$.

8) T F If $(1,1,1)$ is a maximum of f under the constraints $g(x,y,z) = c, h(x,y,z) = d$, and the Lagrange multipliers satisfy $\lambda = 0, \mu = 0$, then $(1,1,1)$ is a critical point of f.

9) T F If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.

10) T F Let (x_0,y_0) be a saddle point of $f(x,y)$. For any unit vector $\vec{u}$, there are points arbitrarily close to (x_0,y_0) for which ∇f is parallel to $\vec{u}$.

11) T F A function $f(x,y)$ on the plane for which the absolute minimum and the absolute maximum are the same must be constant.

12) T F The sign of the Lagrange multiplier tells whether the critical point of $f(x,y)$ constrained to $g(x,y) = 0$ is a local maximum or a local minimum.

13) T F The point $(0,1)$ is a local minimum of the function $x^3 + (\sin(y-1))^2$.

14) T F The integral $\int_0^1 \int_0^{\pi/4} \int_0^{2\pi} 1 \; d\theta d\phi d\rho$ is the volume of the ice cream cone obtained by intersecting $x^2 + y^2 \leq z^2$ with $x^2 + y^2 + z^2 \leq 1$.

15) T F The formula $\int_0^1 \int_0^y f(x,y) \; dxdy = \int_0^1 \int_0^x f(x,y) \; dydx$ holds for all functions $f(x,y)$.

16) T F The surface area of a surface does not depend on the parameterization of the surface.

17) T F The formula for surface area is $\int \int_R |\vec{r}_u \cdot \vec{r}_v| \; dudv$.

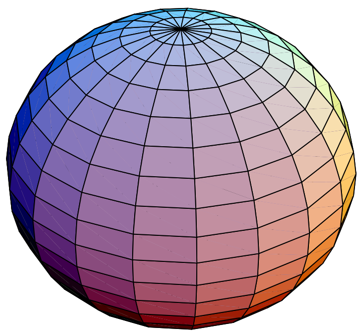
18) T F The integral $\int_0^1 \int_0^1 f(x,y) \; dxdy$ is the volume under the graph of f and so non-negative.

19) T F $\int_{-2}^2 \int_{-3}^3 (x^2 + y^2) \sin(y) \; dxdy = 0$.

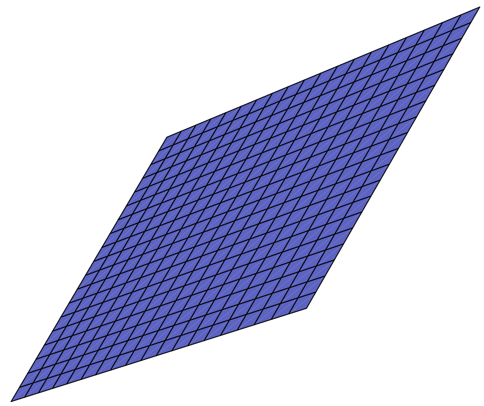
20) T F When changing to cylindrical coordinates, we include a factor $\rho^2 \sin(\phi)$.

Problem 2) (10 points)

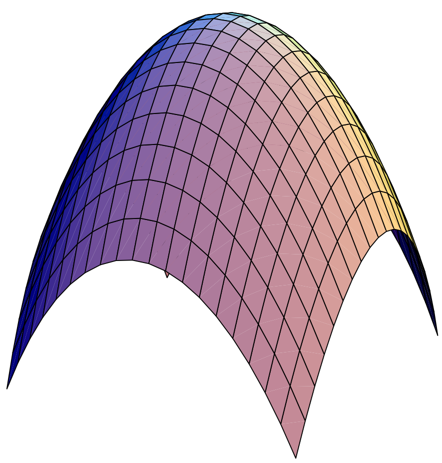
Match the parametric surfaces $S = \vec{r}(R)$ with the corresponding surface integral $\int\int_S \; dS = \int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv$. No justifications are needed.



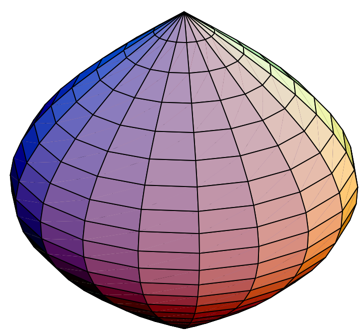
I



II



III



IV

Enter I,II,III,IV here	Surface integral
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^1 \int_0^1 \sqrt{1+4u^2+4v^2} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^1 \int_0^1 \sqrt{3} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^{2\pi} \int_0^{\pi} \sin(v)\sqrt{1+\cos(v)^2} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^{2\pi} \int_0^{\pi} \sin(v) \; dvdu$

Problem 3) (10 points)

Find and classify all the critical points of the function $f(x, y) = xy(4 - x^2 - y^2)$.

Problem 4) (10 points)

Find the area of the moon-shaped region outside the disc of radius 1 and inside the Cardiod $r = 1 + \cos(\theta)$.

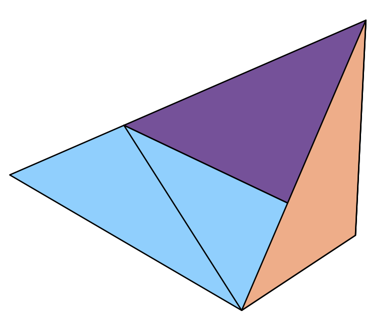
Problem 5) (10 points)

Minimize the function $E(x, y, z) = \frac{k^2}{8m}(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2})$ under the constraint $xyz = 8$, where k^2 and m are constants.

Remark. In quantum mechanics, E is the ground state energy of a particle in a box with dimensions x, y, z. The constant k is usually denoted by $\hbar$ and called the Planck constant.

Problem 6) (10 points)

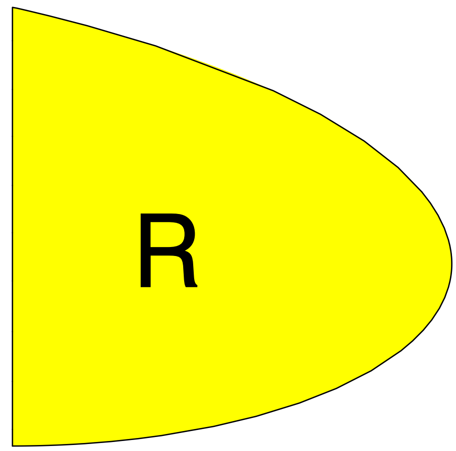
A beach wind protection is manufactured as follows. There is a rectangular floor $ACBD$ of length a and width b. A pole of height c is located at the corner C and perpendicular to the ground surface. The top point P of the pole forms with the corners A and C one triangle and with the corners B and C an other triangle. The total material has a fixed area of $g(a, b, c) = ab + ac/2 + bc/2 = 12$ square meters. For which dimensions a, b, c is the volume $f(a, b, c) = abc/6$ of the tetrahedral protected by this configuration maximal?



Problem 7) (10 points)

A region R in the xy-plane is given in polar coordinates by $r(\theta) \leq \theta$ for $\theta \in [0, \pi/2]$. Find the double integral

$$\iint_R \frac{\sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}(\pi/2 - \sqrt{x^2+y^2})} \, dx \, dy .$$



Problem 8) (10 points)

Find the volume of the solid bound by $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ above the cone $x^2 + y^2 = z^2$ and in the first octant $x \geq 0, y \geq 0, z \geq 0$.

Problem 9) (10 points)

Find $\int \int \int_R z^2 \, dV$, where R of the solid obtained by intersecting $\{1 \leq x^2 + y^2 + z^2 \leq 4\}$ with the double cone $\{z^2 \geq x^2 + y^2\}$.

Problem 10) (10 points)

Consider the region inside $x^2 + y^2 + z^2 = 2$ above the surface $z = x^2 + y^2$.

a) Sketch the region.

b) Find its volume.