4/18/2006 SECOND HOURLY **Math 21a, Spring 2006**

Name:

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MWF 10 Joachim Krieger
MWF 11 Matt Leingang
MWF 11 Veronique Godin
TTH 10 Oliver Knill
TTH 115 Thomas Lam

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Justify all your answers for problems 4-9. As usual, the path to the answer is always graded too.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- All unspecified functions are differentiable as many times as necessary.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F $\int\int\int_R 1\ dxdydz$ is the volume of R.

2) T F At a critical point of a function f, the gradient vector has length 1.

3) T F At a critical point (x,y) of a function f, the tangent plane to the graph of f does not exist.

4) T F For any point (x,y) which is not a critical point, there is a unit vector $\vec{u}$ for which $D_{\vec{u}}f(x,y)$ is nonzero.

5) T F If $f_{xx}(0,0) = 0, D = f_{xx}f_{yy} - f_{xy}^2 \neq 0$, and $\nabla f(0,0) = \langle 0,0 \rangle$, then $(0,0)$ is a saddle point.

6) T F A continuous function defined on the closed unit disc $x^2 + y^2 \leq 1$ has an absolute maximum inside the disc or on the boundary.

7) T F The function $f(x,y) = x^2 - y^2$ has a neither a local maximum nor a local minimum at $(0,0)$.

8) T F If (x,y) is a maximum of $f(x,y)$ under the constraint $g(x,y) = 5$ then it is also a maximum of $f(x,y) + g(x,y)$ under the constraint $g(x,y) = 5$.

9) T F The functions $f(x,y)$ and $g(x,y) = (f(x,y))^6$ always have the same critical points.

10) T F For $f(x,y,z) = x^2 + y^2 + 2z^2$, the vector $\nabla f(1,1,1)$ is perpendicular to the surface $f(x,y,z) = 4$ at the point $(1,1,1)$.

11) T F $f(x,y) = \sqrt{16 - x^2 - y^2}$ has both an absolute maximum and an absolute minimum on its domain of definition.

12) T F If (x_0,y_0) is a critical point of $f(x,y)$ and $f_{xy}(x_0,y_0) < 0$, then (x_0,y_0) is a saddle point of f.

13) T F The vector $\vec{r}_v(u,v)$ of a parameterized surface $(u,v) \mapsto \vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ is always perpendicular to the surface.

14) T F Suppose f has a maximum value at a point P relative to the constraint $g = 0$. If the Lagrange multiplier $\lambda = 0$, then P is also a critical point for f without the constraint.

15) T F At a saddle point, all directional derivatives are zero.

16) T F The minimum of $f(x,y)$ under the constraint $g(x,y) = 0$ is always the same as the maximum of $g(x,y)$ under the constraint $f(x,y) = 0$.

17) T F At a local maximum (x_0,y_0) of $f(x,y)$, one has $f_{yy}(x_0,y_0) \leq 0$.

18) T F The volume of a sphere of radius 1 is equal to the volume under the graph of $f(x,y) = \sqrt{1 - x^2 - y^2}$ inside the unit disc $x^2 + y^2 \leq 1$.

19) T F $\int_0^1 \int_0^1 (x^2 + y^2)\ dxdy = 2/3$.

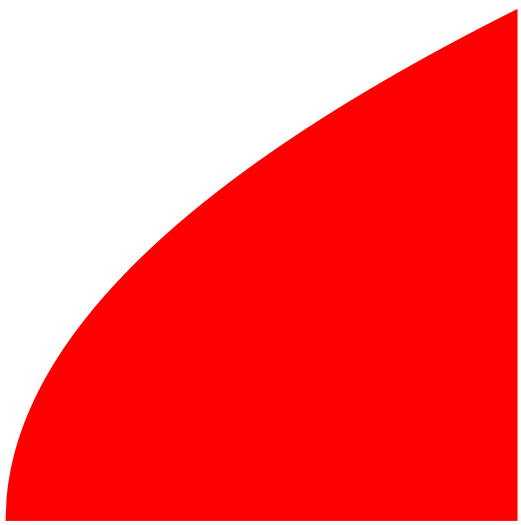
20) T F The function $f(x,y) = \int_0^x \int_0^y g(u) + g(v)\ dudv$ has the critical points (t,t), where t is a root of g.

Problem 2) (10 points)

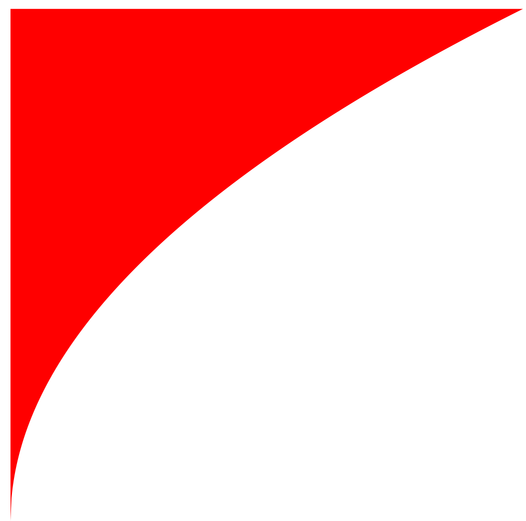
Match the regions R with the corresponding double integral $\int\int_R f(x,y)\,dxdy$. No justifications are needed for this problem.

I

II



III



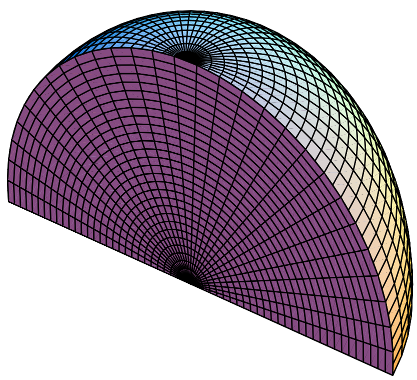
IV

Enter I,II,III,IV here	Integral
	$\int_0^1 \int_{\sqrt{x}}^1 f(x,y)\,dydx$
	$\int_0^1 \int_{x^2}^1 f(x,y)\,dydx$
	$\int_0^1 \int_0^{\sqrt{x}} f(x,y)\,dydx$
	$\int_0^1 \int_0^{x^2} f(x,y)\,dydx$

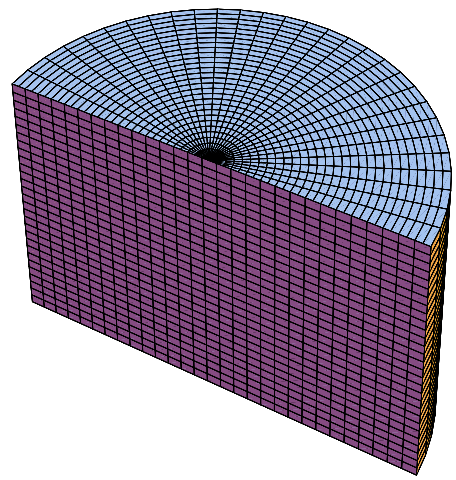
Enter I,II,III,IV here	Integral
	$\int_0^1 \int_{y^2}^1 f(x,y)\,dxdy$
	$\int_0^1 \int_0^{y^2} f(x,y)\,dxdy$
	$\int_0^1 \int_{\sqrt{y}}^1 f(x,y)\,dxdy$
	$\int_0^1 \int_0^{\sqrt{y}} f(x,y)\,dxdy$

Problem 3) (10 points)

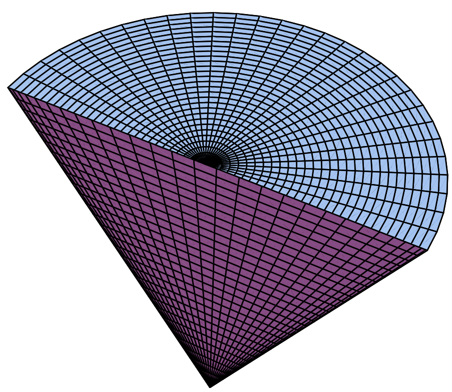
Match the solids E with the corresponding triple integral $\int\int\int_E f\ dV$. There is one triple integral, for which no picture of the solid is given. Mark this triple integral with O. No justifications are needed for this problem.



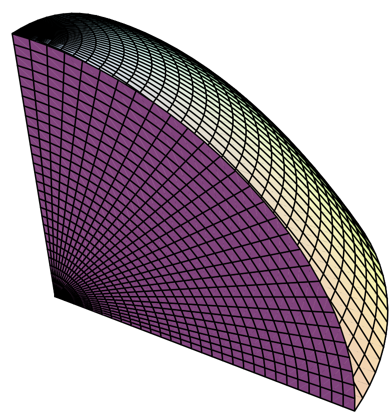
I



II



III



IV

Enter O, I,II,III or IV here	Integral
	$\int_0^1 \int_0^{\pi/2} \int_0^{1-x^2-y^2} f(x,y,z)\ r dz d\theta dr$
	$\int_0^1 \int_0^{\pi} \int_0^{\pi/2} \rho^2 \sin(\phi) f(\rho,\theta,\phi)\ d\phi d\theta d\rho$
	$\int_0^1 \int_0^{z} \int_0^{\pi/2} f(r,\theta,z)\ r\ d\theta dr dz$
	$\int_0^1 \int_0^{z} \int_0^{\pi} f(r,\theta,z)\ r\ d\theta dr dz$
	$\int_0^1 \int_0^{\pi} \int_0^{1} f(r,\theta,z)\ r\ dr d\theta dz$

Problem 4) (10 points)

Find all the critical points of the function $f(x,y) = xy^3 - \frac{x^2}{2} - \frac{3y^2}{2}$.
For each critical point, specify whether it is a local maximum, a local minimum or a saddle point and show how you know.

Problem 5) (10 points)

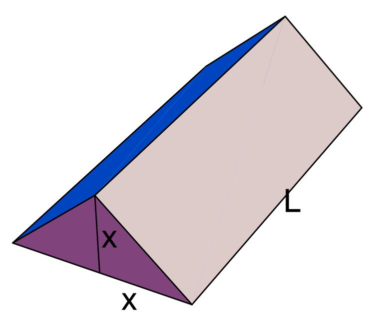
a) (6 points) Find all critical points of $f(x,y) = 3xe^y - e^{3y} - x^3$ and classify them.

b) (4 points) Does the function have a absolute maximum or absolute minimum? Make sure to justify also this answer.

Problem 6) (10 points)

We minimize the surface of a roof of height x and width $2x$ and length $L = \sqrt{2}y$ if the volume $V(x,y) = x^2\sqrt{2}y$ of the roof is fixed and equal to $\sqrt{2}$. In other words, you have to minimize $f(x,y) = 2x^2 + 4xy$ under the constraint $g(x,y) = x^2y = 1$. Solve the problem with the Lagrange method.

Note: this problem can also be solved by substituting one of the variables in the constraints. If done properly, such a solution needs more work. **No credit** is given for such a substitution solution in this problem.



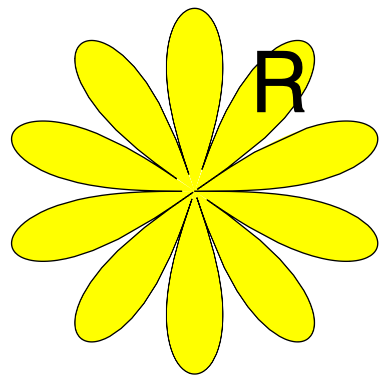
Problem 7) (10 points)

Evaluate the double integral

$$\int_0^{27} \int_{x^{1/3}}^3 \frac{1}{1+y^4} \, dydx \; .$$

Problem 8) (10 points)

The flower type region R is given in polar coordinates as $0 \leq \theta \leq 2\pi, 0 \leq r \leq \sqrt{|\sin(5\theta)|}$. Compute the moment of inertia $I = \int \int_R r^2 \, dA$ of this region.



Problem 9) (10 points)

A solid E in space is given by the inequalities $x^2 + y^2 + z^2 \leq 1$, $x^2 + y^2 \geq z^2$. (In other words, the solid is obtained by cutting away the double cone $x^2 + y^2 \leq z^2$ from the unit ball $x^2 + y^2 + z^2 \leq 1$.) Compute the triple integral

$$\int \int \int_E (x^2 + y^2 + z^2) \, dxdydz \; .$$