3/14/2006 FIRST HOURLY FIRST PRACTICE **Math 21a, Spring 2006**

Name:

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
9		10
Total:		110

Problem 1) TF questions (20 points) No justifications needed

1) T F The length of the sum of two vectors is always the sum of the length of the vectors.

2) T F For any three vectors, $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$.

3) T F The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.

4) T F The functions $\sqrt{x + y - 1}$ and $\log(x + y - 1)$ have the same domain of definition.

5) T F If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R.

6) T F The line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ hits the plane $2x + 3y + 4z = 9$ at a right angle.

7) T F The graph of $f(x, y) = \cos(xy)$ is a level surface of a function $g(x, y, z)$.

8) T F For any two vectors, $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$.

9) T F If $|\vec{v} \times \vec{w}| = 0$ for all vectors $\vec{w}$, then $\vec{v} = \vec{0}$.

10) T F If $\vec{u}$ and $\vec{v}$ are orthogonal vectors, then $(\vec{u} \times \vec{v}) \times \vec{u}$ is parallel to $\vec{v}$.

11) T F Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$.

12) T F The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$.

13) T F The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid.

14) T F If $\vec{v} \times \vec{w} = (0, 0, 0)$, then $\vec{v} = \vec{w}$.

15) T F Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$.

16) T F Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$.

17) T F The function $u(x, t) = x^2/2 + t$ satisfies the heat equation $u_t = u_{xx}$.

18) T F Any function of three variables $f(x, y, z)$ satisfies the partial differential equation $f_{xyz} + f_{yzx} = 2f_{zxy}$.

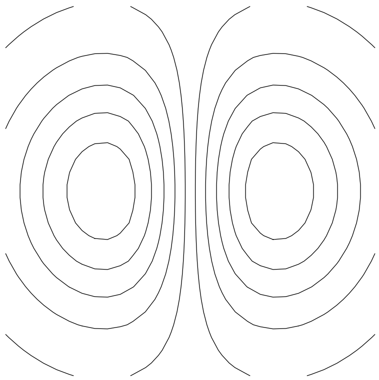
19) T F If $f_x(x, y) = f_y(x, y)$ for all x, y, then $f(x, y)$ is a constant.

20) T F The value of the function $f(x, y) = \sin(-x + 2y)$ at $(0.001, -0.002)$ can by linear approximation be estimated as -0.003.

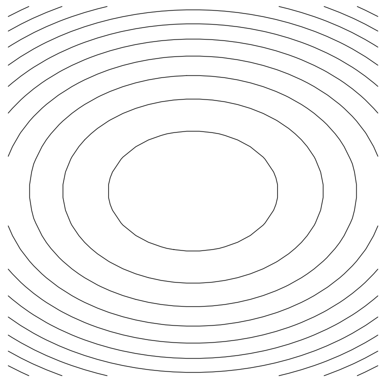
Problem 2a) (5 points)

Match the contour maps with the corresponding functions $f(x, y)$ of two variables. No justifications are needed.

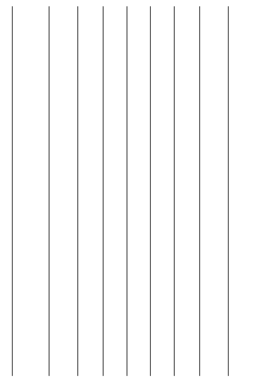
I



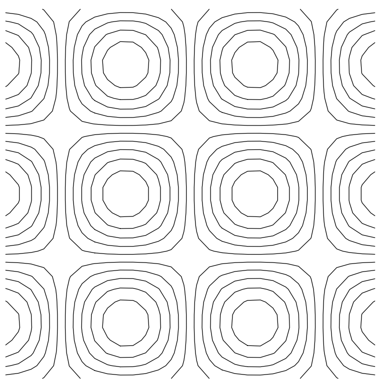
II



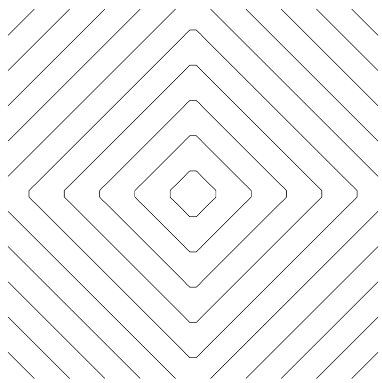
III



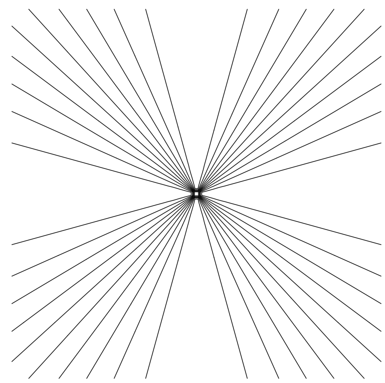
IV



V



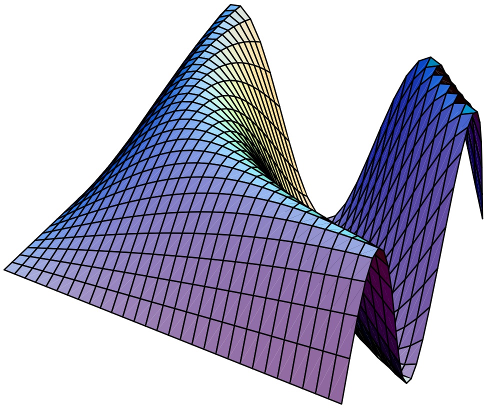
VI



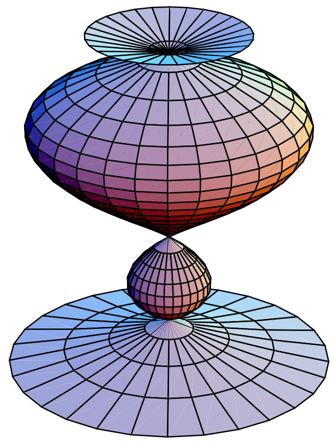
Enter I,II,III,IV,V or VI here	Function $f(x, y)$
	$f(x, y) = \sin(x)$
	$f(x, y) = x^2 + 2y^2$
	$f(x, y) = \|x\| + \|y\|$
	$f(x, y) = \sin(x)\cos(y)$
	$f(x, y) = xe^{-x^2-y^2}$
	$f(x, y) = x^2/(x^2 + y^2)$

Problem 2b) (5 points)

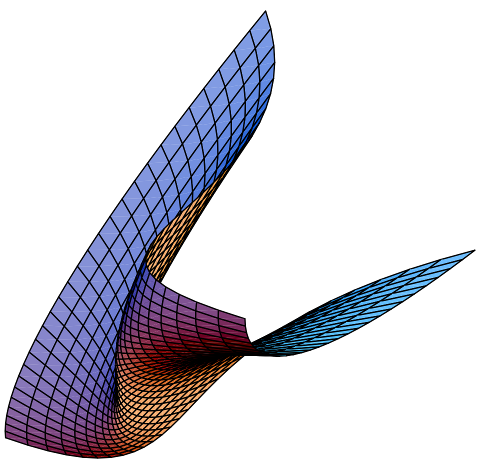
Match the parametric surfaces with their parameterization. No justification is needed.



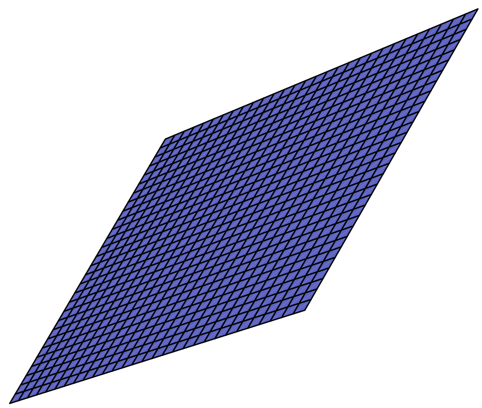
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$(u, v) \mapsto (u, v, u + v)$
	$(u, v) \mapsto (u, v, \sin(uv))$
	$(u, v) \mapsto (0.2 + u(1 - u^2))\cos(v), (0.2 + u(1 - u^2))\sin(v), u)$
	$(u, v) \mapsto (u^3, (u - v)^2, v)$

Problem 3) (10 points)

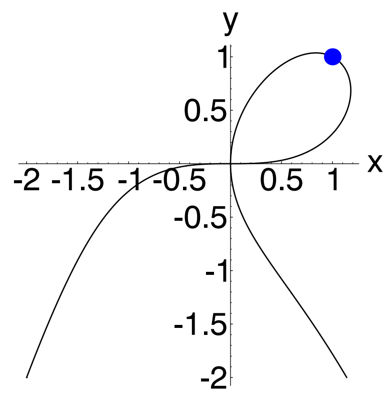
Use the technique of linear approximation to estimate $f(\log(2) + 0.001, 0.006)$ for $f(x, y) = e^{2x-y}$. (Here, log means the natural logarithm).

Problem 4) (10 points)

Consider the equation

$$f(x,y) = 2y^3 + x^2y^2 - 4xy + x^4 = 0$$

It defines a curve, which you can see in the picture. Near the point $x = 1, y = 1$, the function can be written as a graph $y = y(x)$. Find the slope of that graph at the point $(1,1)$.



Problem 5) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 6) (10 points)

a) (4 points) Find the area of the parallelogram with vertices $P = (1,0,0)$ $Q = (0,2,0)$, $R = (0,0,3)$ and $S = (-1,2,3)$.

b) (3 points) Verify that the triple scalar product has the property $[\vec{u}+\vec{v}, \vec{v}+\vec{w}, \vec{w}+\vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$.

c) (3 points) Verify that the triple scalar product $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq ||\vec{u}|| \cdot ||\vec{v}|| \cdot ||\vec{w}||$$

Problem 7) (10 points)

Find the distance between the two lines

$$\vec{r_1}(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r_2}(t) = \langle 1 + t, t, t \rangle \ .$$

Problem 8) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

Problem 9) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
b) (3 points) Set up the integral for the arc length of one of the curves.
c) (3 points) What is the arc length of this curve?

Problem 10) (10 points)

a) (6 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$ at the point $\vec{r}(0)$.

b) (4 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(5t), \sin(5t), -10t \rangle$ at the point $\vec{r}(0)$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} ,$$

where $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$. The curvatures in b) can be derived from the curvature in a). There is no need to redo the calculation, but we need a justification.