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| MWF 10 Samik Basu |
| MWF 10 Joachim Krieger |
| MWF 11 Matt Leingang |
| MWF 11 Veronique Godin |
| TTH 10 Oliver Knill |
| TTH 115 Thomas Lam |

- Please mark the box to the left which lists your section and make sure you have written down your name in the box above.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which the grader can not read will not receive credit.
- Except for the TF and matching problem, show your work.
- No notes, books, calculators, computers, or other electronic aids can be used.
- You have 90 minutes time to complete your work.

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| 1 | | 20 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| 9 | | 10 |
| Total: | | 100 |

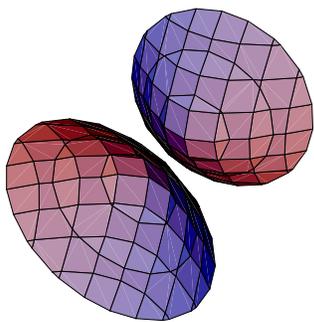
Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

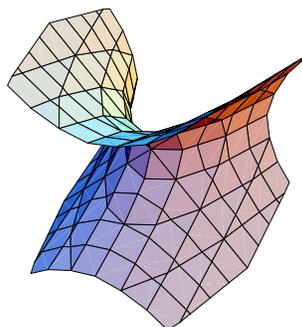
- 1) T F The vectors $\langle 1, 2, 1 \rangle$ and $\langle 3, 2, -5 \rangle$ are perpendicular.
- 2) T F $|\vec{v} \times \vec{w}| = |v||w| \cos(\alpha)$, where α is the angle between \vec{v} and \vec{w} .
- 3) T F The vector $\vec{i} \times (\vec{j} \times \vec{k})$ has length 1.
- 4) T F The distance between the z -axis and the line $x - 1 = y = 0$ is 1.
- 5) T F If two vectors \vec{v} and \vec{w} are perpendicular, then the lengths of $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$ are the same.
- 6) T F If $\vec{r}_1(t)$ is a parameterization of a curve and $\vec{r}_2(t)$ is a second parameterization of the same curve and $\vec{r}_1(0) = \vec{r}_2(0)$, then the velocity vectors $\vec{r}_1'(t)$ and $\vec{r}_2'(t)$ are the same.
- 7) T F There is a surface which has both hyperbola and parabola as traces.
- 8) T F The parameterization $\vec{r}(\theta, \phi) = \langle 5 \cos(\theta) \sin(\phi), 2 \sin(\theta) \sin(\phi), 3 \cos(\phi) \rangle$ with $\theta \in [0, 2\pi), \phi \in [0, 2\pi]$ describes an ellipsoid.
- 9) T F If the velocity vector $\vec{r}'(t)$ is perpendicular to the vector $\vec{r}(t)$, then the parametrized curve $\vec{r}(t)$ is on a sphere.
- 10) T F The function $f(x, y) = 2x^2y^2/(x^2 + y^2)$ with $f(0, 0) = 0$ is continuous in the entire plane.
- 11) T F The function $f(x, y) = x^{\sin(x)} + \cos(xy^6)$ satisfies the partial differential equation $f_{xyxy}(x, y) = f_{xxyy}(x, y)$ everywhere in the plane.
- 12) T F The surface $f(x, y, z) = x^2 + y^2 - z^2 = -1$ is a one-sheeted hyperboloid.
- 13) T F The curvature of the circle $x^2 + y^2 = 4$ is 2.
- 14) T F The equation $x^2 + y^2/4 = 1$ in space describes an ellipsoid.
- 15) T F For any two vectors \vec{v}, \vec{w} we have $\text{proj}_{\vec{v}}(\vec{w}) = \text{proj}_{\vec{w}}(\vec{v})$.
- 16) T F The set of points in space which have distance 1 from a line form a cylinder.
- 17) T F The surface given in spherical coordinates as $\theta = \pi/3$ is a half cone.
- 18) T F The velocity vector of a parametric curve $\vec{r}(t)$ always has constant length.
- 19) T F If f satisfies the PDE $f_{xx} = f_{tt}$, then $g(t, x) = f(t + x, t - x)$ satisfies the PDE $g_{tx} = 0$.
- 20) T F The volume of a parallelepiped spanned by $\vec{u}, \vec{v}, \vec{w}$ is $|(\vec{u} \times \vec{v}) \cdot \vec{w}|$.

Problem 2) (10 points)

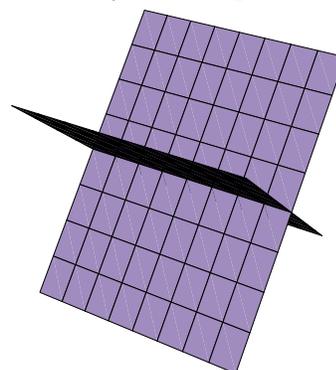
Match the equation with the pictures. No justifications are necessary in this problem.



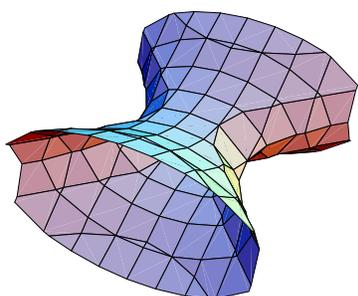
I



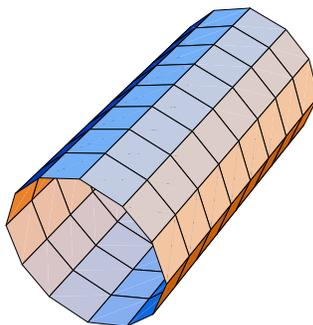
II



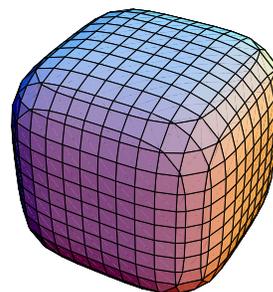
III



IV



V



VI

| Enter I,II,III,IV,V,VI here | Equation |
|-----------------------------|----------------------------|
| | $x^4 + y^4 + z^4 - 1 = 0$ |
| | $-x^2 + y^2 - z^2 - 1 = 0$ |
| | $x^2 + z^2 = 1$ |
| | $-y^2 + z^2 = 0$ |
| | $x^2 - y^2 + 3z^2 - 1 = 0$ |
| | $x^2 - y - z^2 = 0$ |

Problem 3) (10 points)

a) (5 points) Find the distance of the point $P = (1, 2, 3)$ to the plane $x + y + z = 1$.

b) (5 points) Find the distance of the point $P = (1, 2, 3)$ to the line $x - 1 = y - 3 = z - 4$.

Problem 4) (10 points)

a) (4 points) Write down the parameterization of the sphere $(x - 1)^2 + (y - 1)^2 + (z + 2)^2 = 9$.

using suitably centered spherical coordinates θ and ϕ .

b) (3 points) The latitudes on the sphere are curves defined by the equation $\phi = \text{constant}$. Write down the parametric equations for the latitude $\phi = \pi/4$.

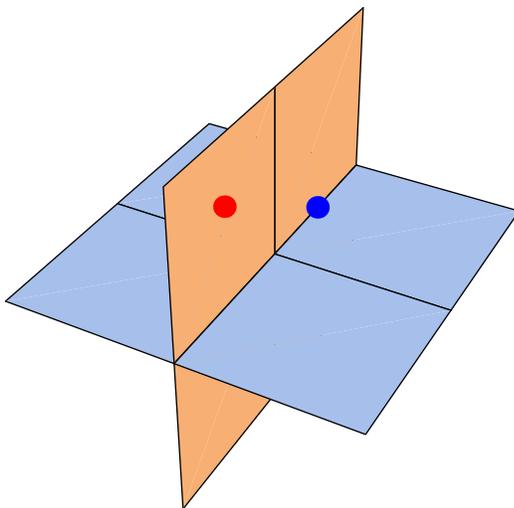
c) (3 points) Write down the arc length integral for this curve and evaluate it.

Problem 5) (10 points)

Given the plane $x + y + z = 6$ containing the point $P = (2, 2, 2)$. Given is also a second point $Q = (3, -2, 2)$.

a) (5 points) Find the equation $ax + by + cz = d$ for the plane through P and Q which is perpendicular to the plane $x + y + z = 6$.

b) (5 points) Find the symmetric equation for the intersection of these two planes.



Problem 6) (10 points)

Intersecting the elliptic cylinder $x^2 + y^2/4 = 1$ with the plane $z = \sqrt{3}x$ gives a curve in space.

a) (3 points) Verify that this curve is parametrized by $\vec{r}(t) = \langle \sin(t), 2 \cos(t), \sqrt{3} \sin(t) \rangle$ and give the parameter interval.

b) (3 points) Compute the unit tangent vector \vec{T} to the curve at the point $(0, 2, 0)$.

c) (4 points) Write down the arc length integral and evaluate the arc length of the curve.

Problem 7) (10 points)

We know the acceleration $\vec{r}''(t) = \langle 2, 1, 3 \rangle + t\langle 1, -1, 1 \rangle$ and the initial position $\vec{r}(0) = \langle 0, 0, 0 \rangle$ and initial velocity $\vec{r}'(0) = \langle 0, 0, 0 \rangle$ of an unknown curve $\vec{r}(t)$. Find $\vec{r}(100)$.

Problem 8) (10 points)

The elliptic paraboloid $f(x, y, z) = x^2 + 2y^2 - z = 0$ contains the point $(1, 1, 3)$.

a) (4 points) Find the equation for the tangent plane at $(1, 1, 3)$.

b) (3 points) Write down the linear approximation function $L(x, y, z)$ of $f(x, y, z)$ at $(1, 1, 3)$.

c) (3 points) Estimate $f(1.01, 1.0002, 2.999)$.

Problem 9) (10 points)

a) (5 points) Show that the function $u(t, x) = \cos(2t - x) + \sin(2t - x)$ satisfies the **Klein-Gordon partial differential equation**

$$u_{tt} = u_{xx} - 3u .$$

b) (3 points) Describe the level curves of u .

c) (2 points) Parameterize one of the level curves $u(t, x) = 1$.