1 Checklist Week 1:

I Definitions

coordinates, vectors, sphere, dot product, cross product triple cross product, parallel vectors, orthogonal vectors, scalar projection $\text{Comp}_v(w)$ and vector projection $\text{Proj}_v(w)$.

II Facts

$v.w = v_1w_1 + v_2w_2 + v_3w_3 = |v||w|\cos(\alpha)$
$|v \times w| = |v||w|\sin(\alpha)$ is area of paralellogram
$|u \cdot (v \times w)|$ volume of parallelepiped

III Algorithms

Adding, subtracting and scaling vectors geometrically as well as algebraically. Completion of square. Compute dot, cross, triple products. Find distance between points. Find vector orthogonal to two vectors. Area of parallelogram, volume of a parallelepiped spanned by three vectors.

2 Checklist Week 2:

I Definitions

Line	plane
$\frac{(x-x_0)}{a} = \frac{(y-y_0)}{b} = \frac{z-z_0}{c}$	$ax + by + cz = d$
$\vec{x} = \vec{x}_0 + t\vec{v}$	$\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}.$

Domain and range of functions $f(x,y)$. Graph $G = \{(x, y, f(x,y))\}$. **Intercepts**: intersections of G with coordinate axes. **Traces**: intersections with coordinate planes. **Generalized traces:** intersections with $\{x = c\}$, $\{y = c\}$ or $\{z = c\}$. **Quadric:** Ellipsoid, Paraboloid, One and Two sheeted Hyperboloid, Cylinder, Cone, Hyperbolic Paraboloid.

II Facts

Plane $ax + by + cz = d$ has normal vector $\vec{n} = (a, b, c)$.
Line $\frac{(x-x_0)}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$ contains $\vec{v} = (a, b, c)$. Sets $g(x, y, z) = c$ describe **surfaces**.
Special case: **graphs** $g(x, y, z) = z - f(x, y)$.
Linear equation (i.e. $2x + 3y + 5z = 7$) defines planes.
Quadratic equation (i.e. $x^2 - 2y^2 + 3z^2 = 4$) defines quadric.

III Algorithms

Using dot and cross product to derive distance formulas: distance point-plane, distance point-line, line-line. Geometric constructions: example: finding plane through P, Q, R. Intersection of two planes, intersection of line-plane, angles between lines and placnes, switch from different descriptions of lines and planes. Sketch and match graphs of $f(x, y)$. Sketch and match quadrics. Completion of squares to find type of quadric.

3 Checklist Week 3:

Plane and space curves $\vec{r}(t)$
Velocity $\vec{r}\,'(t)$, Acceleration $\vec{r}\,''(t)$
Position from velocity $\vec{r}(t) = \int_0^t \vec{r}'(s)\,ds + \vec{r}(0)$.
Unit tangent $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$.
Unit normal $\vec{N}(t) = \vec{T}'(t)/|\vec{T}'(t)|$
Binormal vector $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$.
Curvature $\kappa(t) = |\vec{T}'(t)|/|\vec{r}\,'(t)|$
Arc length $\int_a^b |\vec{r}\,'(t)|\,dt$.

II Facts
$\vec{r}\,'(t)$ is tangent to the curve.
$\vec{v} = \vec{r}\,'$ then $\vec{r} = \int_0^t \vec{v}\,dt + \vec{c}$.
$\kappa(t) = \frac{|r\,'(t) \times r\,''(t)|}{|r\,'(t)|^3}$.
Identities like $\frac{d}{dt}(\vec{v}(t) \cdot \vec{w}(t)) = \vec{v}'(t) \cdot \vec{w}(t) + \vec{v}(t) \cdot \vec{w}'(t)$.
T, N, B are unit vectors which are perpendicular to each other.

III Algorithms
Compute $|\vec{r}\,'(t)|, \vec{T}(t)$ for curve $\vec{r}(t)$.
Draw curves in the plane or in space.
Match curves with their parametric equations.
Find parameterizations of curves (i.e. intersections of surfaces).

4 Checklist Week 4:

I Definitions
polar $(x, y) = (r\cos(\theta), r\sin(\theta))$.
cylindrical $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$
spherical $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$
$g(r, \theta) = 0$ **polar curve**, especially $r = f(\theta)$, polar graphs.
$g(r, \theta, z) = 0$ **"cylindrical surface"**, especially $r = f(z, \theta)$ or $r = f(z)$ **surface of revolution**.
$g(\rho, \theta, \phi) = 0$ **"spherical surface"** especially $\rho = f(\theta, \phi)$.
$\vec{r}(u, v) = (x(u, v), y(u, v), z(u, v))$ parametrized surface. Fix one variable: grid curves.

II Examples
$x^2 + y^2 = r^2$, $\vec{r}(t) = (r\cos(t), r\sin(t))$.
$x^2 + y^2 + z^2 = \rho^2$, $\vec{r}(\theta, \phi) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$.
$ax + by + cz = d$, $\vec{r}(u, v) = \vec{r}_0 + u\vec{v} + v\vec{w}, (a, b, c) = \vec{v} \times \vec{w}$.
$r(\theta, z) = g(z)$, $\vec{r}(u, v) = (g(v)\cos(u), g(v)\sin(u), v)$.
$g(x, y, z) = z - f(x, y) = 0$, $\vec{r}(u, v) = (u, v, f(u, v))$.

III Algorithms
plot curves and surfaces from implicit or parametric equation.
match curves and surfaces with equations.
parametrize curves and surfaces.
parametric description $\vec{r}(u, v) \leftrightarrow$ implicit description $g(x, y, z) = 0$ for planes, sphere, graphs, surfaces of revolution.
translate from and into polar coordinates.
translate from and into spherical or cylindrical coordinates.

5 Checklist Week 5:

I Definitions
$f_x(x,y) = \frac{\partial}{\partial x} f(x,y)$ **partial derivative**
partial differential equation PDE like $F(f, f_x, f_t, f_{xx}, f_{tt}) = 0$
The function f is the **unknown**.

- $f_t = f_{xx}$ heat equation
- $f_{tt} - f_{xx} = 0$ wave equation
- $i\hbar f_t + f_{xx} = 0$ Schrödinger

- $f_x - f_t = 0$ transport equation
- $f_x f - f_t = 0$ Burgers equation
- $f_{xx} + f_{yy} = 0$ Laplace equation

$L(x,y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$
linear approximation.

Tangent line: $L(x,y) = L(x_0, y_0)$ or
$ax + by = d$ with $a = f_x(x_0, y_0), b = f_y(x_0, y_0), d = ax_0 + by_0$
Tangent plane: $L(x,y,z) = L(x_0, y_0, z_0)$ or
Can **estimate** $f(x,y,y)$ by $L(x,y,z)$ near (x_0, y_0, z_0).
$f(x,y)$ **differentiable** that is if f_x, f_y continuous.

II Facts

$f_{xy} = f_{yx}$ **Clairot's theorem**.

$\vec{r}_u(u,v), \vec{r}_v$ are tangent to surface.

III Algorithms
Distinguish ODE's and PDE's.
Verify that given function satisfies a PDE.
Compute tangent lines and tangent planes.
Estimate $f(x_0 + dx, y_0 + dy)$ for small dx, dy.
Decide about differentiability of $f(x,y)$.

6 Checklist Week 6:

I Definitions
$\nabla f(x,y) = (f_x, f_y)$, $\nabla f(x,y,z) = (f_x, f_y, f_z)$, **gradient**
∇ spelled "Nabla". $D_{\vec{v}} f = \nabla f \cdot \vec{v}$ **directional derivative** (we define it like this for all $\vec{v}$!!!)
$L(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0)$.
linearization.

II Facts
$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$ chain rule.

$\nabla f(\vec{x}_0)$ orthogonal to level set $f(\vec{x}) = c$ containing $\vec{x}_0$.

$\frac{d}{dt} f(\vec{x} + t\vec{v}) = D_v f(\vec{x}$ by chain rule.
$(\vec{x} - \vec{x}_0) \cdot \nabla f(\vec{x}) = 0$ **tangent space** at $\vec{x}_0$
Directional derivative maximal in $\vec{v} = \nabla f$ direction.
Partial derivatives are special directional derivatives.
If $D_v f(\vec{x}) = 0$ for all $\vec{v}$, then $\nabla f(\vec{x}) = \vec{0}$.

III Algorithms
Implicit differentiation: example: $f(x, y(x)) = 0$, compute $y'(x) = -f_x/f_y$ without knowing $y(x)$: use chain rule $f_x 1 + f_y y'(x) = 0$.
Compute directional derivatives $D_{\vec{v}} f$.
Find direction where directional derivative is maximal.
Find tangent lines and tangent planes. Estimate $f(x,y)$ near a point $f(x_0, y_0)$ as $L(x,y)$, where $L(x,y)$ is the linearization of f.

7 Checklist Week 7:

$\nabla f(x, y) = (0, 0)$, **critical point** or **stationary point**

$D = f_{xx}f_{yy} - f_{xy}^2$ **discriminant** or **Hessian determinant**

$f(x_0, y_0) \geq f(x, y)$ in a neighborhood of (x_0, y_0) **local maximum**

$f(x_0, y_0) \leq f(x, y)$ in a neighborhood of (x_0, y_0) **local minimum**

$f(x_0, y_0) \geq f(x, y)$ for all x, y, global maximum or absolute maximum

$f(x_0, y_0) \leq f(x, y)$ for all x, y, global minimum or absolute minimum

$\nabla f(x, y) = \lambda \nabla g(x, y), g(x, y) = c, \lambda$ **Lagrange multiplier**.

Two constraints: $\nabla f = \lambda \nabla g + \mu \nabla h, g = c, h = d$.

II Facts

Solutions to $\nabla f(x, y) = \lambda \nabla g(x, y), g(x, y) = c$ are candidates for maxima or minima of f under the constraint $g(x, y) = c$.

> **Second derivative test:**
> $\nabla f = (0, 0), D > 0, f_{xx} < 0$ local maximum
> $\nabla f = (0, 0), D > 0, f_{xx} > 0$ local minimum
> $\nabla f = (0, 0), D < 0$ saddle point

III Algorithms

Find and classify critical points in some domain.

Find maxima and minima for constrained optimization problems.

Find maxima and minima on a region with boundary (unconstrained problem inside the region, Lagrange at the boundary).

8 Checklist Week 8:

I Definitions

$\int \int_R f(x, y) \, dA$ **double integral**

$\int_a^b \int_c^d f(x, y) \, dydx$ integral over **rectangle**

$\int_a^b \int_{c(x)}^{d(x)} f(x, y) \, dydx$ **type I region**

$\int_c^d \int_{a(y)}^{b(y)} f(x, y) \, dxdy$ **type II region**

II Facts

$\int_a^b \int_c^d f(x, y) \, dydx = \int_c^d \int_a^b f(x, y) \, dxdy$ **Fubini**

$\int \int_R 1 \, dxdy$ area of region R.

$\int \int_R f(x, y) \, dxdy = \int \int_U f(r \cos(\theta), r \sin(\theta)) \boxed{r} \, drd\theta$ polar.

$\int \int_R f(x, y) \, dxdy$ volume of solid bounded above by graph(f) xy-plane.

III Algorithms

Evaluate double integrals.

Calculate areas of regions.

Calculate volume of certain of bodies.

Switch order of integration.

Change from rectangular to polar coordinates.

Calculate areas of parameterized surfaces.

9   Checklist Week 9:

I Definitions

$\int\int_R |\vec{r}_u \times \vec{r}_v|\, dudv$ **surface area**
$\int\int\int_R f(x,y,z)\, dV$ **triple integral**
$\int_a^b \int_c^d \int_u^v f(x,y,z)\, dydx$ integral over **rectangular box**
$\int\int\int_R f(r,\theta,z)\; \boxed{r}\;\; dzdrd\theta$ **cylindrical coordinates**
$\int\int\int_R f(\rho,\theta,z)\; \boxed{\rho^2 \sin(\phi)}\;\; dzdrd\theta$ **spherical coordinates**

II Facts

$V = \int\int\int_R 1\, dV$ **volume** of solid R.
$M = \int\int\int_R \rho(x,y,z)\, dV$ **mass** of solid R with density ρ.
$(\int\int\int_R x\, dV/V, \int\int\int_R y\, dV/V, \int\int\int_R z\, dV/V)$ **center** of mass.

III Algorithms

Switch order of integration in triple integrals.
Apply the right coordinate system for integration.
Make a picture of the region one has to integrate over.

II Facts

$\int_a^b \int_c^d f(x,y)\, dxdy = \int_c^d \int_a^b f(x,y)\, dxdy$ **Fubini**
Formulas for surface area in case of sphere, surface of revolution and graph.

III Algorithms
Change order of integration.
Use correct coordinate system.
Get from integral the region and from the region the integral.

10   Checklist Week 10:

I Definitions
$\vec{F}(x,y) = \langle P(x,y), Q(x,y)\rangle$ vector field in 2D,
$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$ vector field in 3D,
$\int_\gamma F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt$ line integral $\vec{F}(x,y) = \nabla f(x,y)$ gradient field = potential field = conservative, f is called the **potential** of F.

II Facts

Line integrals are independent of the parametrization.
Line integrals are positive if the angle between the velocity vector and the field is acute at all points of the curve.
Line integrals have the interpretation of "Work", if F is interpreted as a force field. If $Q_x - P_y \neq 0$ at some place, then $F = (P,Q)$ is not a gradient field $F = (f_x, f_y)$. (Clairot).

III Algorithms

Match vector fields with formulas.
Compute line integrals for a given parametrized curve and a given vector field.
See, whether a field is not a gradient field, by checking with Clairot.

11 Checklist Week 11:

$F(x, y) = (P, Q)$, $\text{curl}(F) = Q_x - P_y$. $\text{div}(F) = P_x + Q_y$.

$F(x, y, z) = (P, Q, R)$, $\text{curl}(P, Q, R) = (R_y - Q_z, P_z - R_x, Q_x - P_y)$, $\text{div}(P, Q, R) = P_x + Q_y + R_z$.

$\Delta f = \text{divgrad}(f) = f_{xx} + f_{yy} + f_{zz}$ Laplacian for functions

NABLA calculus:

$$\boxed{\begin{array}{l} \nabla = (\partial_x, \partial_y, \partial_z), \ \text{grad}(f) = \\ \nabla f, \ \text{curl}(\vec{F}) = \nabla \times \vec{F}, \\ \text{div}(f) = \nabla \cdot \vec{F}. \end{array}} \quad \boxed{\begin{array}{l} \text{div}(F) = \nabla \cdot F \\ \Delta f = \nabla \cdot \nabla f \end{array}}$$

II Facts

Fundamental theorem of line integrals: $F(x, y) = \nabla f(x, y)$

$$\boxed{\int_a^b F(r(t)) \cdot r'(t) \, dt = f(r(b)) - f(r(a))}$$

F defined in in a simply connected region R $\int_\gamma F \, dr = 0$, for every closed curve γ.

Green's theorem: C boundary of R, (R is "to the left")

$$\boxed{\int_C F \cdot dr = \int \int_R \text{curl}(F) \, dxdy}$$

$$\boxed{\begin{array}{l} \text{div}(\text{curl}(F)) = 0. \\ \text{curl}(\text{grad}(F)) = \vec{0} \\ \text{curl}(\text{curl}(F)) = \text{grad}(\text{div}(F)) - \Delta(F). \end{array}}$$

III Algorithms

Find curl and divergence for specific vector fields F. Decide whether a vector field F can be of the form $F = \text{curl}(G)$. Decide whether a vector field F can be of the form $F = \text{grad}(f)$. Use FTL to switch between line integrals and summation of end points. Use Greens theorem to switch between double and line integrals. Calculate with ∇ (Nabla calculus).

12 Checklist Week 12:

I Definitions

$F(x, y, z)$ vector field, $S = r(R)$ parametrized surface

$r_u \times r_v$ normal vector, $\vec{n} = \frac{r_u \times r_v}{|r_u \times r_v|}$ unit normal vector

$|r_u \times r_v| dudv = dS$ surface element

$r_u \times r_v dudv = d\vec{S} = \vec{n} dS$ normal surface element.

$\int \int_S f \, dS = \int \int_S f(r(u, v)) |r_u \times r_v| \, dudv$ **surface integral**

$\int \int_S F \, d\vec{S} = \int \int_S F(r(u, v)) \cdot (r_u \times r_v) \, dudv$ **flux integral**.

II Facts

Stokes's theorem: C boundary of surface S

$$\boxed{\int_C F \cdot dr = \int \int_S \text{curl}(F) \cdot dS}$$

Gauss (Divergence) theorem: S boundary of region E

$$\boxed{\int \int_S F \cdot dS = \int \int \int_E \text{div}(F) \, dV}$$

III Algorithms

Use Stokes theorem to switch between flux and line integrals.

Use Gauss theorem to switch between triple and flux integrals.