Answers to Second Math 21a Practice Hourly 1

Note: Problems 1-6 count 8 points each, while Problem 7 counts for 2 points.

1. Answer:
   a) $|\mathbf{v}| = \sqrt{21}$, $|\mathbf{w}| = 1$, $|\mathbf{v} - 3\mathbf{w}| = \sqrt{6}$.
   b) This number is $\mathbf{v} \cdot \mathbf{w} = 4$.
   c) (-2, 0, 1).
   d) $b = -2$ and $c = 0$.

2. Answer:
   a) (3/2, 0, 0), (0, -3/2, 0) and (0, 0, 3). (There are infinitely many other possibilities)
   b) (2, -2, 1), or any non-zero multiple
   c) The distance is 1.
   d) Parametric: Send $t \to (3t, 3t, 3)$. Nonparametric: $x = y$ & $z = 3 - 2x + 2y$.
   (There are infinitely many other possibilities.)

3. Answer:
   a) $(0, -3, 4\pi)$.
   b) $2\sqrt{\pi}\,(-3, 0, 4)$
   c) $t \to (-3t, -3, 4(\pi + t))$.
   d) The velocity vector at general t is $2t\,(3\cos(t^2), -3\sin(t^2), 4)$ whose length is $10t$.
   Thus, the distance is the integral of this last function from 0 to $\sqrt{\pi}$ which is $5\sqrt{\pi}$.

4. Answer:
   a) $\mathbf{p}$ is on L and $\mathbf{v}$ is tangent to L.
   b) $t \to \mathbf{p} + t\,\mathbf{v}$.
   c) (-4, -3, 0); the case of $t = -1$ in the preceding parameterization.
   d) $d = |\mathbf{p} \times \mathbf{v}|/|\mathbf{v}| = \sqrt{\frac{34}{35}}$.

5. Answer:
   a) The plane where $x - 3z = 0$. (There are infinitely many other possibilities.)
   b) $\mathbf{w} \cdot \mathbf{v} = 0$ requires $b = 16$.
   c) $b = -6$ and $c = -2$.
   d) If $\mathbf{s}$ is parallel to $\mathbf{v}$, then $\mathbf{u} \times \mathbf{s}$ will be perpendicular to $\mathbf{v}$. For this, take $c = -6$.

6. Answer:

a) True. Indeed, since $\mathbf{r}\cdot\mathbf{r} = |\mathbf{r}|^2$ and $\frac{d}{dt}(\mathbf{r}\cdot\mathbf{r}) = 2\,\mathbf{r}\cdot\frac{d}{dt}\mathbf{r}$, which is zero, the distance of the particle to the origin stays constant. Thus, it moves on the surface of a sphere.

b) False: If the particle motion is given by $\mathbf{r}(t) = t\,\mathbf{k}$, then it moves on a line and not a sphere.

c) True: See the preceding answer. In fact, any $\mathbf{r}(t)$ of the form $\mathbf{r}(t) = \mathbf{r}_0 + t\,\mathbf{k}$ will do.

d) True: Write $\mathbf{r}(t) = (a(t), b(t), c(t))$, so $\mathbf{k}\times\mathbf{r} = (-b(t), a(t), 0)$ and $\mathbf{k}\times\frac{d}{dt}\mathbf{r} =$ $(-\frac{d}{dt}b(t), \frac{d}{dt}a(t), 0)$. As this is orthogonal to $\mathbf{k}\times\mathbf{r}$, so $a\frac{d}{dt}a + b\frac{d}{dt}b = 0$, which says that $2^{-1}\frac{d}{dt}(a^2 + b^2) = 0$ so $a^2 + b^2$ is constant. Thus, the x and y coordinates of the particle move on a circle while the z coordinate can do what it likes. This puts the motion on a circular cylinder.

7. Answer: 0. The vector $\mathbf{u}\times\mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.