Trigonometric Integration Review

Useful Properties

1. $\sin(-\theta) = -\sin(\theta)$
2. $\cos(-\theta) = \cos(\theta)$
3. $\cos^2\theta + \sin^2\theta = 1$
4. $1 + \tan^2\theta = \sec^2\theta$ (Divide 1 by $\cos^2\theta$)
5. $\cot^2\theta + 1 = \csc^2\theta$ (Divide 1 by $\sin^2\theta$)

6. $\cos(\theta_1 \pm \theta_2) = \cos(\theta_1)\cos(\theta_2) \mp \sin(\theta_1)\sin(\theta_2)$
5. $\sin(\theta_1 \pm \theta_2) = \sin(\theta_1)\cos(\theta_2) \pm \cos(\theta_1)\sin(\theta_2)$
6. $\cos(2\theta) = \cos(\theta + \theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
7. $\sin(2\theta) = \sin(\theta + \theta) = 2\cos(\theta)\sin(\theta)$

8. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$ (Rearrange 6)
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ (Rearrange 6)
10. $\cos(\frac{\theta}{2}) = \sqrt{\frac{1+\cos(\theta)}{2}}$ (Substitute θ from 8 with $\frac{\theta}{2}$ and take square root)
11. $\sin(\frac{\theta}{2}) = \sqrt{\frac{1-\cos(\theta)}{2}}$ (Substitute θ from 8 with $\frac{\theta}{2}$ and take square root)

12. $\sin\theta_1 \sin\theta_2 = \frac{1}{2}\cos(\theta_1 - \theta_2) - \frac{1}{2}\cos(\theta_1 + \theta_2)$ (Subtract the two forms of 4 and divide by 2)
13. $\cos\theta_1 \cos\theta_2 = \frac{1}{2}\cos(\theta_1 - \theta_2) + \frac{1}{2}\cos(\theta_1 + \theta_2)$ (Add the two forms of 4 and divide by 2)
14. $\sin\theta_1 \cos\theta_2 = \frac{1}{2}\sin(\theta_1 - \theta_2) + \frac{1}{2}\sin(\theta_1 + \theta_2)$ (Add the two forms of 5 and divide by 2)

15. $\cos(\theta_1) + \cos(\theta_2) = 2\cos(\frac{\theta_1+\theta_2}{2})\cos(\frac{\theta_1-\theta_2}{2})$ (Substitute θ_1 and θ_2 from 13 with $\frac{\theta_1+\theta_2}{2}$ and $\frac{\theta_1-\theta_2}{2}$)
16. $\cos(\theta_1) - \cos(\theta_2) = -2\sin(\frac{\theta_1+\theta_2}{2})\sin(\frac{\theta_1-\theta_2}{2})$ (Substitute θ_1 and θ_2 from 12 with $\frac{\theta_1+\theta_2}{2}$ and $\frac{\theta_1-\theta_2}{2}$)
17. $\sin(\theta_1) + \sin(\theta_2) = 2\sin(\frac{\theta_1+\theta_2}{2})\cos(\frac{\theta_1-\theta_2}{2})$ (Substitute θ_1 and θ_2 from 14 with $\frac{\theta_1+\theta_2}{2}$ and $\frac{\theta_1-\theta_2}{2}$)
18. $\sin(\theta_1) - \sin(\theta_2) = 2\sin(\frac{\theta_1-\theta_2}{2})\cos(\frac{\theta_1+\theta_2}{2})$ (Substitute θ_2 from 17 $-\theta_2$ and apply 1)

Trigonometric Integration Rule of Thumbs

When you see integrands involving $\sqrt{a^2 - x^2}$, substitute $x = a\sin(\theta)$.

When you see integrands involving $\sqrt{a^2 + x^2}$, substitute $x = a\tan(\theta)$.

Practice Questions

1. Find the area of a circle of radius *r* using: a) single variable integral, b) double integral.
2. Find the volume of a sphere of radius *r* using: a) double integral, b) triple integral.
3. Find the volume of a square pyramid with side *s* and height *h* using: a) double integral, b) triple integral.