HW 30 Solutions

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§11.8 # 1, 3, 7, 10, 21

(1) maximum value ≈ 60

minimum value ≈ 30

The max and min occur when the level curves of the function $f(x,y)$ have a common tangent line with the constraint $g(x,y)=8$. (in other words, their gradient vectors are parallel).

(3) maximize / minimize $f(x,y) = x^2 - y^2$ such that $x^2+y^2=1$

$\nabla f(x,y) = \lambda \nabla g(x,y)$ and $g(x,y)=1$

$\nabla f(x,y) = (2x, -2y)$ $\quad\quad$ $\nabla g(x,y) = (2x, 2y)$

$$\left.\begin{array}{ll} \text{i)} & 2x = \lambda \cdot 2x \\ \text{ii)} & -2y = \lambda \cdot 2y \\ \text{iii)} & x^2+y^2=1 \end{array}\right\}$$

case 1: $\lambda = 0$

$\Rightarrow$ $2x=0$ and $-2y=0$

$\Rightarrow$ $x=y=0$

but this contradicts equation (iii)

case 2: from (i) we get $x=0$ or $\lambda=1$

if $x=0$: from (iii) we get $y=\pm 1$

if $\lambda=1$: from (ii) we get $y=0$

then from (iii) we get $x=\pm 1$

four possible cases: $(0,1), (0,-1), (1,0), (-1,0)$

$f(0,1) = -1$ $\quad\quad$ $f(1,0)=1$

$f(0,-1) = -1$ $\quad\quad$ $f(-1,0)=1$

thus, $\boxed{\begin{array}{l} \text{maximum value } f(\pm 1, 0) = 1 \\ \text{minimum value } f(0,\pm 1) = -1 \end{array}}$