Math 1b
Calculus, Series, and Differential Equations

Harvard University

Fall 2005

Welcome to Math 1b! We hope that this term will be full of stimulating mathematics for you.

Course Head

Dr. Matthew Leingang, Preceptor in Mathematics
Office: Science Center 323
Office Hours: Tuesdays, Wednesdays, 1:00–3:00pm.
leingang@math.harvard.edu

Course Content and Goals

"The book of the universe is written in the language of mathematics." Galileo wrote this four hundred years ago, even before Newton and Leibniz discovered calculus. The statement is as valid today as ever: We use functions in all the sciences, and calculus allows us to analyze the functions and draw scientific conclusions.

Math 1b is a second semester calculus course for students who have previously been introduced to the basic ideas of differential and integral calculus. Over the semester we will study three (related) topics, topics that form a central part of the language of modern science:

- applications and methods of integration,
- differential equations,
- infinite series and the representation of functions by power series.

The material we take up in this course has applications in physics, chemistry, biology, environmental science, astronomy, economics, statistics, and just about everything else. We want you to leave the course not only with computational ability, but with the ability to use these notions in their natural scientific contexts, and (we hope) with an appreciation of their mathematical beauty and power.

Integration

In your previous math courses you studied differential calculus and were introduced to integral calculus. You studied the Fundamental Theorem of Calculus which illuminates the connection between differentiation and integration. We will start the semester continuing with integration. You should already be familiar with the definite integral, its definition as the limit of Riemann sums, and its calculation using antiderivatives and simple u-substitution. In Math 1b we will learn more powerful techniques of antidifferentiation. The goal is not to transform you into an integration automaton (we live in the computer age), but to have you acquire familiarity with the techniques and the ability to apply them to some standard situations.

Speaking of standard situations, the definite integral enables us to tackle a multitude of problems in a wide array of fields; we will use the notion of Riemann sums—dividing, approximating, and summing—to apply integration in various contexts. More important than any one particular application is the ability to apply the integration as appropriate in problem solving; we will devote time to developing your skill in doing this.

Differential Equations

We will continue with *differential equations*, equations modeling rates of change. Differential equations permeate quantitative analysis throughout the physical and social sciences. A scientific principle is often modeled by a differential equation. Solving these exactly is often complicated and very often impossible; but in all cases there are techniques to analyze solutions.

Infinite Series

In the final unit of the course we will look at infinite sums (called *infinite series*). You already are aware that a rational number such as $\frac{1}{3}$ can be approximated by a non-repeating decimal, such as $0.33\ldots$ or $0.3333\ldots$ and represented by an infinite sum, such as $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \ldots$. Irrational numbers such as e and π not only can be approximated by non-repeating decimals, but have representations as infinite sums as well. In this unit we begin by approximating familiar functions like e^x , e^{-x^2}, $\sin x$ and $\cos(x^2)$ by polynomials.

The functions listed are challenging to evaluate and some are challenging to integrate. By using polynomials of increasingly large degree we can get increasingly good approximations to the functions. In fact, we will find that many functions, such as e^x and $\sin x$ can be represented by infinite polynomials known as *power series*. We will learn to compute, understand, and manipulate these representations. Polynomial approximations based on these power series representations are widely used by engineers, physicists, and many other scientists. They can even help us solve differential equations whose solutions had eluded us.

Course Requirements

You may find this course to involve a bit more work and move more quickly than your last. This is intentional but not malicious. A little mind-flexing is necessary to make sure you learn not only the formulas and the mechanics but the principles that allow us to apply mathematics to other disciplines.

Class and Problem Sessions

Math 1b is taught in sections of about 25 students that meet three hours per week. The section leader is either a teaching fellow or faculty member in mathematics. The course head sets the syllabus for all sections, and teaches a section as well. The department believes this teaching setup (as opposed to large lectures) is more conducive to an interactive learning environment. This means you will have the opportunity to participate in class rather than simply have to absorb it as if it were on television. Small sections are there for your benefit; take advantage of it! Ask questions, get to know your section leader, and be a part of your education.

Each section has a Course Assistant, who will be in class, collect and correct homework assignments, and hold weekly problem sessions. Problem sessions are extremely valuable; they will be generally be devoted to working problems and amplifying the class material, and understanding the homework assignments. A schedule of all problem sessions will be posted on the course web site; feel free to go to any course assistants problem session.

Sections can meet MWF 9, 10, 11, and 12, and TTH 10 and 11:30. The MWF 9 and 12 sections will only happen with sufficient enrollment (for some reason, a 9:00 section is rather rare). Sections are assigned by a computer program which accounts for your preferences and your conflicts. You will need to sign up for sections by computer *no later than 1:00pm Wednesday, September 21.* If you know how to get to a UNIX prompt you need only type

```
%ssh section@ulam.fas.harvard.edu
```

(the percent sign is the prompt—don't type it) and the program will start. If the last sentence is Greek to you there are many instructions on our department's sectioning page. See below for the address.

Sections will begin Monday or Tuesday, September 26 or 27. There are no classes during the first week other than the introductory meeting.

Homework

Problems are an integral part of the course; it is virtually impossible to learn the material and to do well in the course without working through the homework problems in a thoughtful manner. Think about homework as practice, rehearsal, or training; its major goal is to improve your abilities so you can demonstrate them on exams. Dont just crank through computations and write down answers; think about the problems posed, the strategy you employ, the meaning of the

computations you perform, and the answers you get. It is often in this reflection that the greatest learning takes place.

An assignment will be given at each class meeting. Unless otherwise specified, the assignment is due at the following class meeting and will be returned, graded, at the subsequent class. If you miss a class, then you are responsible for obtaining the assignment and handing it in on time. Solutions put together by the course assistants will be available on the course website. When your homework assignments are returned to you, you can consult the solutions for help with any mistakes you might have made. In order to be fair to fellow students and to the course assistants grading the assignments, problem sets must be turned in on time. We understand that situations arise when homework loses priority, and so when computing your final homework grade, your lowest two (TTH sections) or three (MWF sections) problem set scores will be dropped.

For Tuesday/Thursday sections the problem sets due Tuesday will be larger than those due Thursday. But over the course of the term all sections will have the same problems assigned.

Homework problems will sometimes look a bit different from problems specifically explicitly discussed in class. This is not a failure of your TF; it is a goal of the course to build on class instruction and learn to struggle and succeed at mathematical thinking. In this course as in life there won't always be a recipe to follow when solving problems.

Feel free to use a calculator or computer to check or investigate problems for homework. However, an answer with the explanation "because my calculator says so" will not receive credit. Use the calculator as a learning tool, not as a crutch. *Calculators will not be allowed on examinations, so make sure you are as comfortable working without it as with it.*

You are welcome to collaborate with other students on solving homework problems; in fact, you are encouraged to do so. However, write-ups you hand in must be your own work, you must be comfortable explaining what you have written, and there must be a written acknowledgement of collaboration with the names of you coworkers. This is in accordance with the principles of academic honesty.

Tip: Odd-numbered problems are answered in the back of the textbook. They generally will not be assigned, but you can still use them as tests of your own understanding of the material. Often times, a even-numbered problem will have a similar odd problem near it and this can help.

Pre-Class reading

We will generally require that you read the material in the textbook before we discuss it in class, as opposed to only reading it after the class. After reading and before class, you will answer a short survey on the course web site. There will usually be two questions for each reading. One will be asking what you found most difficult in the reading. The other will be specific to the reading topic. Both will be graded mostly on effort.

The purpose of these assignment is twofold: You will get an extra pass through the material so that by the time you sit down to do the homework you have seen it at least twice. Also, your section leader will read your responses before class and use it to prepare the section with your confusions in mind.

Exams

Exams are common (meaning everybody takes them at once) and given in the evenings. Please keep these exam dates free from conflicts:

Exam	Date/Time	Location
Technique Test	October 6, 7:00-8:00pm	Halls A and D[a]
Technique Re-Test[b]	October 13, 7:00-8:00pm	Halls A and D
Midterm I	October 27, 5:00-7:00pm	Emerson 105
Midterm II	December 1, 6:00-8:00pm	Halls A and D

[a]Students whose last names begin with the letters A–J go to Hall A; those whose last names begin with the letters K–Z go to Hall D.

[b]Optional. The higher of your tests is the one which will "count."

The final is scheduled for Saturday, January 14, 2006.

As stated before, calculators will not be allowed on examinations, due in part to equity issues. We will make sure that problems on the exams require minimal calculation to allow you to spend your time demonstrating your mathematical knowledge as opposed to your calculating ability. We expect you to express your ideas, line of reasoning, and answers clearly.

If, for some reason, you have an unmovable conflict with an exam time, you are expected to contact the course head more than one full week in advance so we can make an attempt at accommodation.

Grades

The course requirements are compiled according to the following weighted average:

Technique Test	10%
Pre-Class Reading	5%
Homework	10%
Midterm I	20%
Midterm II	25%
Final	30%
Total	**100%**

This course score will be assigned a letter grade with roughly this correspondence:

90%+	A
80%+	B
65%+	C
50%+	D

"Roughly" means that this scale may be adjusted for equity, making the cutoffs lower (and letter grades higher) than advertised here. Plus and minus refinements of the above grades will also be decided in the final analysis.

Texts

- *Single Variable Calculus: Concepts and Contexts.* 3rd edition, by James Stewart. Brooks/Cole 2005, ISBN 0-534-41022-7. Most of the course material will come from this text.
- There will be supplementary materials from another textbook. These will be made available as a free download from the course web site.
- *Schaum's Outlines: Precalculus* by Fred Safier, ISBN 0-07-057261-5. This is an optional resource for students who would like to brush up on some of the precalculus prerequisites—functions, trigonometry, logarithms, etc. It's slim and cheap.

Prerequisites and Placement

You should have had a good differential calculus class before coming into Math 1b, one that covered the definition of the integral and the Fundamental Theorem of calculus. Math 1a is certainly acceptable. Also, a set of scores on the Harvard math placement exams in which the first number (precalculus) is at least 20 and the second number (differential calculus) is at least 8 gives a Math 1b placement.

If you have taken AP Calculus BC before, you are probably very well-equipped to take this course. Math 1b covers much more than the BC syllabus, though, so be prepared to work hard even though some of this material will be familiar.

If you you have taken AP Calculus AB before, you are also well-equipped to take this course. There will be students here who have seen some things you haven't, but we are committed to teaching the course to all qualified students.

We find that students often come into this course not remembering much precalculus. We understand that many of you took that a long time ago, and so this prerequisite is often relaxed. This means, however, that you assume the responsibility for refreshing yourself on the important aspects of trigonometry and logarithms that will be taken for granted in the course. The precalculus primer may prove to be a resource.

Placement recommendations are just that; you are free to choose your course as you like. Please talk to a calculus advisor (more information below) if you have questions about your specific situation.

Resources

The following sources of math help are available without any appointment. Take advantage of them!

- Problem Sessions: held by course assistants. Your CA will schedule his or her session to best fit the schedules of your section, but you are free to go to any problem session you wish.
- Office Hours: These are times set aside by your section leader to talk informally with students. Your TF will publicize his or her office hours frequently.
- the Math Question Center: in Loker Common 8:00–10:00 pm every night except Friday and Saturday. Course Assistants and mathematics graduate students staff this center. Besides math help, there is usually food.

Course Web Site

The course web site is located at

`http://my.harvard.edu/course/math1b`

Once you are registered for the course, the site will be available through your regular my.harvard portal.

Miscellaneous

There are some events during the first week which you might find beneficial:

Calculus Advising for students still trying to decide on a first course in mathematics is ongoing this week. Please see the schedule at

`http://www.math.harvard.edu/sectioning`

This is also the place to go for step-by-step instructions on how to section by computer.

The Online Placement Exam is available for student who question their placement from the pencil-and-paper math placement exam. To take the exam, visit

`http://math.placement.fas.harvard.edu/`

The exam requires a standards-compliant browser such as Mozilla (not Internet Explorer); there are detailed instructions on how to set it up on the web site.

The Math Warm-Up Series is a set of talks on some of the topics you may have forgotten since you last took math. This year the schedule is:

Date/Time	Location	Topics
September 20, 2:00–3:30pm	Hall A	Trigonometry
September 20, 4:00–5:30pm	Hall D	Logarithms
September 21, 2:00–3:30pm	SC 309	Algebra

For a more information, visit the series web site at

`http://www.math.harvard.edu/mwus`

Conclusion

We hope you take and enjoy Math 1b!