Name: ______________________ ID#: ______________

# Solutions to Final Examination

Math 1b
Calculus, Series, and Differential Equations

January 14, 2006

Please check your section:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ☐ 0.0 | MWF9 | Matt Bainbridge | | ☐ 4.0 | TTH10 | Pan Peng |
| ☐ 1.0 | MWF10 | Dawei Chen | | ☐ 5.0 | TTH11:30 | David Harvey |
| ☐ 1.1 | MWF10 | Angela Vierling-Claassen | | ☐ 5.1 | TTH11:30 | Jesse Kass |
| ☐ 2.0 | MWF11 | Matthew Leingang | | | | |
| ☐ 2.1 | MWF11 | Chun-chun Wu | | | | |
| ☐ 3.0 | MWF12 | Matt Bainbridge | | | | |

Rules:

- This is a three-hour exam.
- Calculators are not allowed.
- Unless otherwise stated, show all of your work. Full credit may not be given for an answer alone.
- You may use the backs of the pages or the extra pages for scratch work. *Do not unstaple or remove pages as they can be lost in the grading process.*
- Please do not put your name on any page besides the first page.

If you like, you may put your ID number on the top of each page you write on.

Hints:

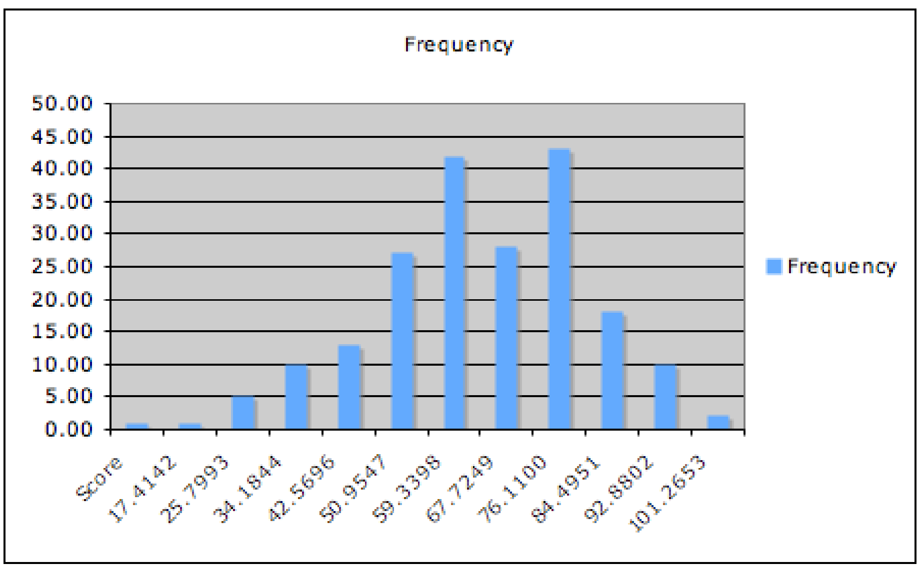
- Read the entire exam to scan for obvious typos or questions you might have.
- Budget your time so that you don't run out.
- Problems may stretch across several pages.
- Relax and do well!

*Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*
—*Handbook for Students*

## Summary Data

| Problem | Points | Mean | Median | Mode | SD | correl |
|---|---|---|---|---|---|---|
| 1 | 15 | 8.48 | 10 | 10 | 4.24 | 0.6160 |
| 2 | 15 | 5.62 | 5 | 0 | 4.59 | 0.6565 |
| 3 | 15 | 11.02 | 14 | 15 | 5.13 | 0.5285 |
| 4 | 12 | 8.26 | 10 | 12 | 3.96 | 0.5629 |
| 5 | 15 | 9.52 | 11 | 13 | 4.69 | 0.6738 |
| 6 | 18 | 9.20 | 10 | 0 | 5.12 | 0.7082 |
| 7 | 10 | 6.06 | 7 | 9 | 3.58 | 0.6444 |
| 8 | 14 | 7.73 | 9 | 0 | 4.59 | 0.7229 |
| Total | 114 | 76.11 | 76.5 | 89 | 16.77 | 1.0000 |

## Histogram



**1.** (15 Points) *Evaluate the following integrals. If the integral is indefinite, give your answer as the most general function of $x$. If the integral is definite, your answer should be a number. If the integral is improper, show your work in evaluating the relevant limit.*

*(i)* $\int_2^3 \frac{\ln x}{x}\,dx$

*Solution.* Let $u = \ln x$, so that $du = \frac{1}{x}\,dx$. Then

$$\int_2^3 \frac{\ln x}{x}\,dx = \int_{\ln 2}^{\ln 3} u\,du = \frac{1}{2}u^2\Big|_{\ln 2}^{\ln 3}$$
$$= \frac{1}{2}\left[(\ln 3)^2 - (\ln 2)^2\right]$$

Common mistakes were algebra ones. Note that

$$\frac{1}{2}(\ln 3)^2 \neq \ln 3$$
$$(\ln 3)^2 - (\ln 2)^2 \neq \left(\frac{\ln 3}{\ln 2}\right)^2$$

Also, since this is a definite integral, the answer is a number and to write "$+C$" at the end is wrong. ▲

*(ii)* $\int x\cos x\,dx$

*Solution.* This requires integration by parts. Let $u = x$ and $dv = \cos x\,\mathrm{dx}$. Then $du = dx$ and $v = \sin x$. So

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

This is an indefinite integral, so the constant of integration must be added. ▲

(continued)

*(iii)* $\displaystyle\int \frac{dx}{x^2-1}$

*Solution.* Using the partial fraction decomposition, we see that

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

So

$$\begin{aligned}\int \frac{dx}{x^2-1} &= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C \\ &= \ln\left|\frac{x-1}{x+1}\right|^{1/2} + C\end{aligned}$$

▲

*(iv)* $\displaystyle\int_0^1 \ln x\,dx$

*Solution.* This integral is improper because $\lim_{x\to-\infty} \ln x = -\infty$. So

$$\int_0^1 \ln x\,dx = \lim_{a\to 0^+}\int_a^1 \ln x\,dx.$$

provided the limit exists. As we showed using integration by parts,

$$\int_a^1 \ln x\,dx = [x\ln x - x]_a^1 = -1 - a\ln a + a.$$

Now

$$\lim_{a\to 0^+}(-1 - a\ln a + a) = -1 - \lim_{a\to 0^+} a\ln a,$$

where the last limit is an indeterminate one of the form $0\cdot\infty$. Using L'Hôpital's Rule, we have

$$\begin{aligned}\lim_{a\to 0^+} a\ln a &= \lim_{a\to 0^+}\frac{\ln a}{1/a} \\ &\overset{\text{H}}{=} \lim_{a\to 0^+}\frac{1/a}{-1/a^2} = \lim_{a\to 0^+}(-a) = 0.\end{aligned}$$

So

$$\int_0^1 \ln x\,dx = -1.$$

Another way to do this is to look at the graph of $y = \ln x$ and realize that

$$\int_0^1 \ln x\,dx = -\int_{-\infty}^0 e^y\,dy = \lim_{a\to-\infty}[-e^y]_a^0 = \lim_{a\to-\infty}[-1+e^a] = -1.$$

▲



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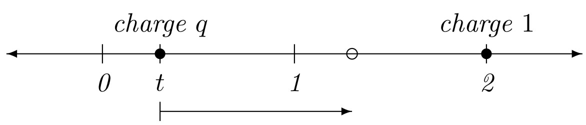
**2.** (15 Points) *Consider two particles which have a positive charge of* $q_1$ *and* $q_2$ *and are a distance* $d$ *apart. Assuming we choose our units conveniently, Coulomb's law states that each particle exerts on the other a force of*

$$F = \frac{q_1 q_2}{d^2}.$$

*(a) (3 points) Consider one particle of charge* $q$ *lying on the* $x$ *axis at position* $x = t$*, where* $0 \leq t \leq 1$ *and another particle of charge* 1 *lying at position* $x = 2$*. Show that the work needed to move the first particle from* $x = t$ *to* $x = t + 1$ *is*

$$W(q) = \left( \frac{1}{1-t} - \frac{1}{2-t} \right) q.$$




*You don't need to derive the Riemann sum for this one; just write down the integral and evaluate it.*

*Solution.* Since work is force times distance traveled when both are constant, we have over a small distance traveled

$$\Delta W = F(x)\, \Delta x.$$

Hence

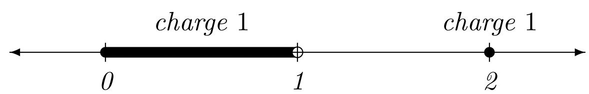
$$W = \int_t^{t+1} F(x)\, dx = \int_t^{t+1} \frac{q}{(x-2)^2}\, dx,$$

which evaluates to the above.

▲

(b) *(3 points) Now consider a metal bar of length one lying along the $x$-axis from $x = 0$ to $x = 1$, and a particle of charge 1 lying at $x = 2$. Suppose the metal bar has a total charge 1 distributed uniformly along the bar. We are going to find the amount of work needed to move the bar to the right by one.*

*Suppose we divide the bar up into small pieces of width $\Delta x$. Approximately how much work will be needed to move that piece of the bar to the right by one?*




*Solution.* Each piece of the bar has charge $\Delta x = \dfrac{1}{n}$ and in its trip of one to the right needs an amount of work equal to

$$\Delta W \approx \left( \frac{1}{1 - x_{i-1}} - \frac{1}{2 - x_{i-1}} \right) \Delta x.$$

▲

(c) *(3 points) Write a Riemann sum which represents the amount of work to move the entire bar one unit to the right. You will need to divide the bar (that is, the interval $[0, 1]$) into $n$ equal pieces. Call them $x_0 = 0, x_1, x_2, \ldots, x_n = 1$. Use your estimate above of the work needed to move the $i$th piece, and add them up.*

*Solution.* So

$$W \approx \sum_{i=1}^{n} \left( \frac{1}{1 - x_{i-1}} - \frac{1}{2 - x_{i-1}} \right) \Delta x.$$

(We can't use right-hand endpoints because the work isn't defined when $x = 1$.) ▲

(d) *(2 points) Write an improper integral which represents the amount of work required to move the bar one unit to the right.*

*Solution.* We change the sum to an integral and get

$$W = \int_0^1 \left( \frac{1}{1-x} - \frac{1}{2-x} \right) dx.$$

▲

(e) *(4 points) Evaluate this improper integral. Is the amount of work required finite or infinite?*

*Solution.* We have

$$\int_0^1 \frac{1}{2-x}\, dx = [-\ln|2-x|]_0^1 = \ln 2,$$

so that's not the problem. However,

$$\begin{aligned}\int_0^1 \frac{1}{1-x}\, dx &= \lim_{b\to 1^+} \int_0^b \frac{1}{1-x}\, dx \\ &= \lim_{b\to 1^+} [-\ln|1-x|]_0^b \\ &= \lim_{b\to 1^+} -\ln|1-x| = \infty\end{aligned}$$

▲



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**3.** (15 Points) *Solve the following initial value problems.*

*(i) (8 points)* $y' - \cos x(y^2 + 1) = 0,\ y(0) = 0.$

*Solution.* We have

$$\frac{dy}{1+y^2} = \cos x\, dx,$$

so

$$\arctan y = \sin x + C.$$

For $y(0)$ to be zero, we must have that $0 = 0 + C$. So

$$y = \tan(\sin x).$$

A common mistake was to treat this equation as a linear one and look for an integrating factor. However, the fact that $y^2$ appears means that this is *not* a linear equation and so this method is invalid. ▲

*(ii) (7 points)* $y'' + 4y = 0,\ y(0) = 2,\ y'(0) = 0.$

*Solution.* Try a solution of the form $y = e^{rt}$. We must have

$$r^2 + 4 = 0 \implies r = \pm 2i.$$

Therefore $y$ takes the form

$$y = A\cos 2t + B\sin 2t.$$

In order for $y$ to satisfy the initial conditions we must have $A = 2$, $B = 0$. Hence $y = 2\cos 2t$.

The most common mistake was to write $r^2 + 4r$ for the characteristic polynomial. That would be the characteristic polynomial of $y'' + 4y' = 0$. It has roots 0 and $-4$, so the characteristic solutions are $e^{-4t}$ and $e^{0t} = 1$. The second most common mistake was to jump to the solutions $e^{-4t}$ and $te^{-4t}$. Those should be the characteristics if the polynomial was $(r+4)^2 = r^2 + 2r + 4$. ▲



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**4.** (12 Points) *Skipatitis B or, as it is more commonly known, Freshman Fatigue, is one of the most common diseases affecting college students. Skipatitis B is caused by a number of enteroviruses in the family Calculaviridaue. It is contagious and is spread through academic interaction with an infected person. Skipatitis B is one of the main killers of Freshman grades. The good news is that after recovering from Freshman Fatigue, a student gains a life-long immunity to the disease. In this problem, we will model the progress of this disease through a college.*

*We say that a person is susceptible if he or she is not yet sick. The number of susceptible people is denoted by the symbol $S$. The number of infected people is denoted by $I$. The symbol $t$ denotes the time elapsed since the disease hit campus.*

*The spead of the disease can be modeled by the following system of differential equations:*

$$\frac{dS}{dt} = -0.0025SI$$
$$\frac{dI}{dt} = 0.0025SI - 0.5I$$

(a) *(2 points) Explain the significance of the minus sign in the equation for $\frac{dS}{dt}$.*

*Solution.* The negative sign, meaning a decrease in the number of susceptible people, represents the people getting sick. This quantity is proportional to the interactions between susceptible people ($S$) and infected people ($I$). A common mistake was to confuse $S$ with $\frac{dS}{dt}$. ▲

(b) *(6 points) Draw the phase plane diagram associated to the system of differential equations modeling the spread of Skipatitis B. Draw the nullclines, indicate with words which are $S$-nullclines and $I$-nullclines, and use arrows to indicate the flow on each nullcline.*

*Solution.* To find the $S$-nullclines, we solve $\frac{dS}{dt} = 0$. This is true when $S = 0$ (the $I$-axis) or $I = 0$ (the $S$-axis).

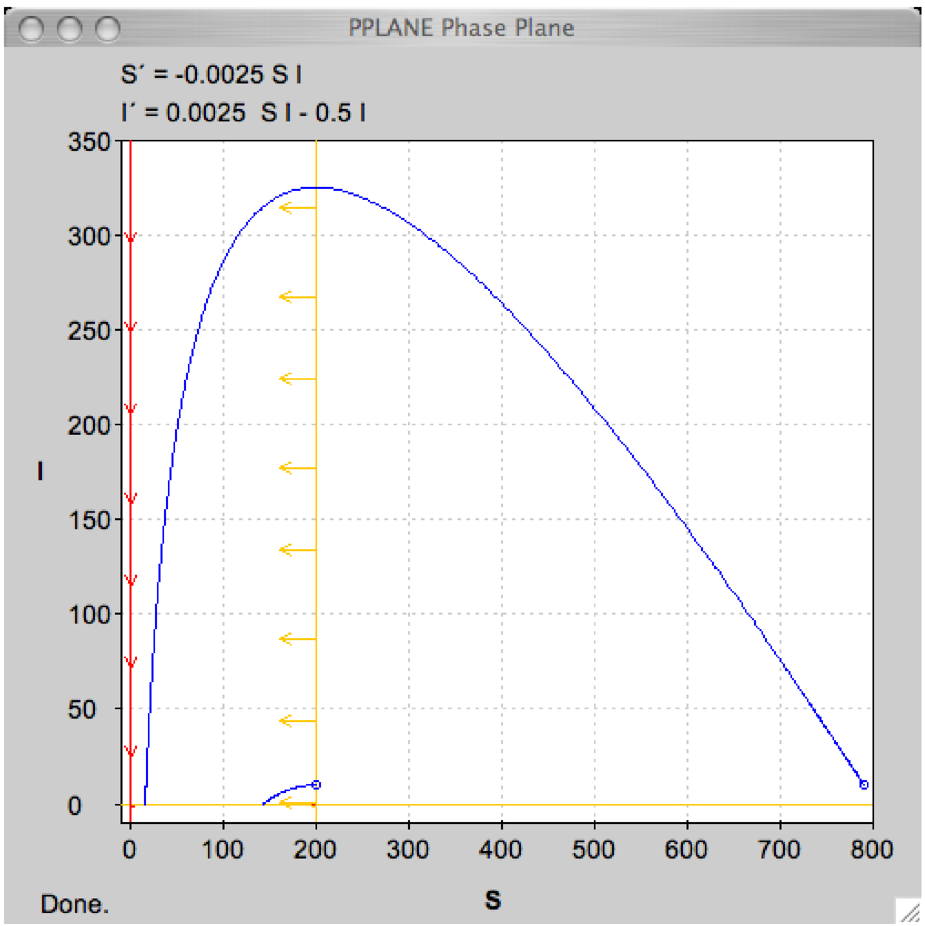
To find the $I$-nullclines, we solve $\frac{dI}{dt} = 0$. This is true when $I = 0$ (the $S$-axis) and when $S = 200$ (a vertical line).

To orient the nullclines, notice that along the $S$-axis, both $\frac{dS}{dt}$ and $\frac{dI}{dt}$ are zero. So each point on the $S$-axis is an equlibrium point. That makes sense, because if there are no infected individuals ($I$=0 on the $S$-axis), then there will be nobody to catch Skipatitis from!

Along the $I$-axis, we have $\frac{dI}{dt} < 0$, so arrows point downward and get larger with increasing $I$. Along the line $S = 200$, $\frac{dS}{dt} < 0$, so arrows point to the left and get larger with increasing $I$.

Here is a phase portrait with two trajectories (see the next part) plotted:

▲




(continued)

(c) *(4 points) School administrators want to prevent Skipatitis from spreading on campus. A student can be vaccinated against the disease by providing the student with a personal tutor. However, this is very expensive.*

*A freshman class of 800 students enter. Ten of them are already infected with Skipatitis while the rest, if not vaccinated, are susceptible. The administrators want to vaccinate as few people as they can, while still preventing the disease from spreading (that is, preventing the infected population from* increasing*).*

*What is the minimum number of people that need to be vaccinated in order to accomplish this goal? Draw trajectories on the phase plane to justify your answer.*

*Solution.* It's important to determine what situation is modeled here and what the question asks. We have $I(0) = 10$ and $S(0) = 790$. Starting there on the phase plane (the large trajectory drawn in), we see the evolution of the system would give us more infected people in the short run.

Vaccinations give us the chance to change $S(0)$. What value would we want to make the number of infected people not increase? Notice from the phase portrait that initial conditions to the *left* of the nullcline at $S = 200$ tend towards equilibrium at $I = 0$. So we need to make sure $S(0) \leq 200$ (small trajectory). If right now $S(0) = 790$, we need to vaccinate at least 590 freshmen. ▲



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**5.** (15 Points) *Determine if the following series converge or diverge.* Explain your reasoning and show your work.

*(i)* $\displaystyle\sum_{n=4}^{\infty} \frac{3}{e^{-n^2}}$

*Solution.* Notice that

$$\lim_{n\to\infty} \frac{3}{e^{-n^2}} = \lim_{n\to\infty} 3e^{n^2} = \infty.$$

So the series cannot converge by the Test for Divergence. ▲

*(ii)* $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

*Solution.* We use the ratio test.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{[(n+1)!]^2}{[2(n+1)]!}}{\frac{(n!)^2}{(2n)!}} = \lim_{n\to\infty} \frac{[(n+1)!]^2}{[n!]^2} \cdot \frac{(2n)!}{(2n+2)!}$$
$$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4}.$$

Since the limit is less than 1, the series converges. ▲

*(iii)* $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$

*Solution.* We can use the Limit Comparison Test with $\frac{1}{n}$. Since

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{2n}{n^2+1}} = \lim_{n\to\infty} \frac{n^2+1}{2n^2} = \frac{1}{2},$$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$ must diverge too. ▲

(continued)

# 5 5

*(iv)* $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^3(n^3)}{n^3}$

*Solution.* Notice that

$$\left|\frac{\sin^3(n^3)}{n^3}\right| \le \frac{1}{n^3},$$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges. So $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^3(n^3)}{n^3}$ converges absolutely. ▲



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**6.** (18 Points)

I. *For which values of $x$ does the power series* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+6)^n}{n2^n}$ *converge? Justify your answer.*

*Solution.* By the ratio test we have

$$\left|\frac{\frac{(-1)^{n+2}(x+6)^{n+1}}{(n+1)2^{n+1}}}{\frac{(-1)^{n+1}(x+6)^n}{n2^n}}\right| = \frac{n}{n+1} \cdot \frac{1}{2}\,|x+6| \longrightarrow \frac{1}{2}\,|x+6|\,,$$

so the series converges if $|x+6| < 2$. So the interval of convergence contains $(-6-2, -6+2) = (-8, -4)$.

At $x = -8$, $x+6 = -2$ and the series reduces to $-\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, which diverges.

At $x = -4$, $x+6 = 2$ and the series reduces to the alternating harmonic series, which converges.

Hence the interval of convergence is $(-8, 6]$. ▲

II. *Suppose the power series* $\displaystyle\sum_{n=0}^{\infty} c_n(x-2)^n$ *converges when $x = 0$ but diverges when $x = -1$. For each of the following values of $x$, determine if the series converges or diverges or if it's impossible to tell. If it is possible to tell, justify your answer.*

(i) $x = 6$

(ii) $x = 3$

(iii) $x = 4$

*Solution.* Let $R$ be the radius of convergence of the power series. Since the series converges at $x = 0$, which is 2 away from 2, we must have $R \geq 2$. Since the series diverges at $x = -1$, we must have $R \leq 3$. Thus the power series must converge absolutely at $x$ if $|x-2| < 2$, and the series must diverge at $x$ if $|x-2| > 3$.

Since 3 is only one away from 2, nearer than the radius of convergence, the series must converge at 3, too. The number 6 is 4 away from 2, definitely outside of the radius of convergence. So the series diverges at 6. At 4 we have no clue, because $R$ could be equal to 2 and convergence could occur at only one of the endpoints (as above) ▲

(continued)

*III. Suppose* $\{a_n\}$ *is a sequence such that* $\sum_{n=0}^{\infty} a_n 3^n$ *converges. Decide if the following series converge or diverge. Justify your answers.*

*(i)* $\sum_{n=0}^{\infty} 2^n a_n$

*Note.* Because you don't know if the terms $\{a_n\}$ are positive, the comparison tests are invalid. Try deciding if the series converges absolutely.

*Solution.* Consider the power series $\sum_{n=0}^{\infty} a_n x^n$. We know this series converges at $x = 3$, so its radius of convergence is at least 3. The series in question is the same power series evaluated at $x = 2$. That is within the radius of convergence, and so this series converges *absolutely* there. ▲

*(ii)* $\sum_{n=0}^{\infty} a_n$

*Solution.* Although it doesn't look like it, this is the same power series evaluated at $x = 1$. So it too converges absolutely. ▲

*(iii)* $\sum_{n=0}^{\infty} \frac{1}{a_n}$

*Solution.* According to the last part, $\sum_{n=0}^{\infty} a_n$ converges, which means that $\lim_{n\to\infty} a_n = 0$. This means that $\lim_{n\to\infty} \frac{1}{a_n}$ does not exist (it may be $\infty$, $-\infty$, or neither depending on $\{a_n\}$). So the series $\sum_{n=0}^{\infty} \frac{1}{a_n}$ cannot converge. ▲



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**7.** (10 Points) *It happens to be true that*

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k-1}=\frac{\pi}{4}.$$

*We will use this to find a fraction that closely approximates* $\frac{\pi}{4}$.

(a) *(3 points) Use the above to give an upper bound to* $\left|\frac{\pi}{4}-\left(1-\frac{1}{3}+\frac{1}{5}\right)\right|$. *This is the error (or remainder) after adding the first three terms of the series. Justify your answer.*

*Solution.* By the remainder estimate for alternating series, the remainder is bounded by the absolute value of the first nonadded term. Hence

$$R_3<\frac{1}{7}.$$

▲

(b) *(3 points) Does the fraction* $1-\frac{1}{3}+\frac{1}{5}=\frac{13}{15}$ *overestimate or underestimate* $\frac{\pi}{4}$*? Remember to justify your answer.*

*Solution.* In a convergent alternating series, the partial sums bounce above and below the sum. Since the first term in this series is positive, it alone is greater than the sum. The second partial sum, that is, the sum of the first two terms, is too small. The third partial sum is too big, i.e., an over estimate for $\frac{\pi}{4}$. ▲

(c) *(4 points) How many terms of the series would you need to add to* be sure *you were approximating* $\frac{\pi}{4}$ *with an error of less than* 0.005*? (This would be two decimal places of accuracy).* No credit will be given for an answer without justification.

*Solution.* We have

$$\mathbb{R}_N\le|a_{N+1}|<\frac{1}{2(N+1)-1}=\frac{1}{2N+1}$$

We want to find $N$ such that $R_N<0.005=\frac{5}{1000}=\frac{1}{200}$. One way to guarantee this is to make sure that

$$\frac{1}{2N+1}<\frac{1}{200}.$$

This would be true if $2N+1>200$. So $N=100$ would suffice. ▲



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**8.** (14 Points)

*I. (3 points) Write down the Maclaurin series for the function*

$$g(x) = \frac{e^x + e^{-x}}{2}.$$

*(The function $g(x)$ is called the* hyperbolic cosine *and is often written* $\cosh x$*.) Express your answer with a single summation sign.*

*Solution.* We have for any $u$ that

$$e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!}.$$

Thus

$$\frac{e^x + e^{-x}}{2} = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right) = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}\right)$$
$$= \frac{1}{2}\sum_{n=0}^{\infty} \frac{[1+(-1)^n]\, x^n}{n!}$$

Now $[1+(-1)^n]$ is 2 or 0, depending on whether $n$ is even or odd. That means the coefficients on all odd powers of $x$ are even. If $n$ is even, let $n = 2k$. We see that

$$\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}.$$

▲

*II. The power series*

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

*converges for all $x$ to $\sin x$.*

*(a) (4 points) Find the Maclaurin series for $f(x) = x^2 \sin(x^2)$.*

*Solution.* We have

$$x^2 \sin x^2 = x^2 \sum_{k=0}^{\infty} \frac{(-1)^k (x^2)^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+4}}{(2k+1)!}$$

▲

(continued)

(b) *(4 points) Find* $f^{(2006)}(0)$.
*Hint.* Use the work you did in part (a)!

*Solution.* We have the power series representation

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+4}}{(2k+1)!},$$

*which must be equal to the Maclaurin series* $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ *for* $f$. Hence the derivative $f^{(2006)}(0)$ is (2006)! times the coefficient of $x^{2006}$ in the series above. But the series above only has nonzero coefficients for multiples of 4. So $f^{(2006)}(0) = 0$. ▲

(c) *(3 points) What is the radius of convergence of the Maclaurin series for* $f$*? Justify your answer.*
*Hint.* You don't have to actually find the series to find its radius of convergence.

*Solution.* The Maclaurin series for $\sin x$ converges for all $x$, and therefore so does that for $\sin(x^2)$. Multiplying by $x$ doesn't change the radius of convergence either. So the series for $x^2 \sin x^2$ converges for all $x$. Thus the radius of convergence of the Maclaurin series for $f$ is $\infty$. ▲



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