6. Let $u = \sin^{-1} x$, $dv = dx \Rightarrow du = \dfrac{dx}{\sqrt{1-x^2}}$, $v = x$. Then

$\int \sin^{-1} x\, dx = x \sin^{-1} x - \int \dfrac{x}{\sqrt{1-x^2}}\, dx$. Setting $t = 1 - x^2$, we get $dt = -2x\, dx$, so

$-\int \dfrac{x\, dx}{\sqrt{1-x^2}} = -\int t^{-1/2}\left(-\frac{1}{2}\, dt\right) = \frac{1}{2}\left(2t^{1/2}\right) + C = t^{1/2} + C = \sqrt{1-x^2} + C$. Hence,

$\int \sin^{-1} x\, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$.

8. First let $u = x^2$, $dv = \sin ax\, dx \Rightarrow du = 2x\, dx$, $v = -\frac{1}{a}\cos ax$. Then by Equation 2,

$I = \int x^2 \sin ax\, dx = -\dfrac{x^2}{a}\cos ax - \int\left(-\dfrac{1}{a}\right)\cos ax\,(2x\, dx) = -\dfrac{x^2}{a}\cos ax + \dfrac{2}{a}\int x\cos ax\, dx$. Next let

$U = x$, $dV = \cos ax\, dx \Rightarrow dU = dx$, $V = \dfrac{1}{a}\sin ax$. So

$\int x\cos ax\, dx = \dfrac{x}{a}\sin ax - \int \dfrac{1}{a}\sin ax\, dx = \dfrac{x}{a}\sin ax + \dfrac{1}{a^2}\cos ax + C_1$. Substituting for $\int x\cos ax\, dx$, we get

$I = -\dfrac{x^2}{a}\cos ax + \dfrac{2}{a}\left(\dfrac{x}{a}\sin ax + \dfrac{1}{a^2}\cos ax + C_1\right) = -\dfrac{x^2}{a}\cos ax + \dfrac{2x}{a^2}\sin ax + \dfrac{2}{a^3}\cos ax + C$.

9. First let $u = (\ln x)^2$, $dv = dx \Rightarrow du = 2\ln x \cdot \frac{1}{x}\, dx$, $v = x$. Then by Equation 2,

$I = \int (\ln x)^2\, dx = x(\ln x)^2 - 2\int x\ln x \cdot \frac{1}{x}\, dx = x(\ln x)^2 - 2\int \ln x\, dx$. Next let $U = \ln x$, $dV = dx \Rightarrow$ $dU = 1/x\, dx$, $V = x$ to get $\int \ln x\, dx = x\ln x - \int x \cdot (1/x)\, dx = x\ln x - \int dx = x\ln x - x + C_1$. Thus, $I = x(\ln x)^2 - 2(x\ln x - x + C_1) = x(\ln x)^2 - 2x\ln x + 2x + C$, where $C = -2C_1$.

11. Let $u = \ln r$, $dv = r^3 dr \Rightarrow du = (1/r)\, dr$, $v = \frac{1}{4}r^4$. Then

$$\int r^3 \ln r\, dr = \tfrac{1}{4}r^4 \ln r - \int \tfrac{1}{4}r^4(1/r)\, dr = \tfrac{1}{4}r^4 \ln r - \tfrac{1}{4}\int r^3 dr = \tfrac{1}{4}r^4 \ln r - \tfrac{1}{4}\left(\tfrac{1}{4}r^4\right) + C$$
$$= \tfrac{1}{4}r^4 \ln r - \tfrac{1}{16}r^4 + C, \text{ or } \tfrac{1}{16}r^4(4\ln r - 1) + C.$$

14. First let $u = e^{-\theta}$, $dv = \cos 2\theta\, d\theta \Rightarrow du = -e^{-\theta}\, d\theta$, $v = \frac{1}{2}\sin 2\theta$. Then

$I = \int e^{-\theta}\cos 2\theta\, d\theta = \frac{1}{2}e^{-\theta}\sin 2\theta - \int \frac{1}{2}\sin 2\theta\left(-e^{-\theta}\, d\theta\right) = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta\, d\theta$.

Next let $U = e^{-\theta}$, $dV = \sin 2\theta\, d\theta \Rightarrow dU = -e^{-\theta}\, d\theta$, $V = -\frac{1}{2}\cos 2\theta$, so

$\int e^{-\theta}\sin 2\theta\, d\theta = -\frac{1}{2}e^{-\theta}\cos 2\theta - \int\left(-\frac{1}{2}\right)\cos 2\theta\left(-e^{-\theta}\, d\theta\right) = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta\, d\theta$. So

$I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\left[\left(-\frac{1}{2}e^{-\theta}\cos 2\theta\right) - \frac{1}{2}I\right] = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}I$

$\Rightarrow \frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1 \Rightarrow$

$I = \frac{4}{5}\left(\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1\right) = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C$.

18. First let $u = x^2 + 1$, $dv = e^{-x}\, dx \Rightarrow du = 2x\, dx$, $v = -e^{-x}$. By (6),

$\int_0^1 (x^2+1)e^{-x}\, dx = \left[-(x^2+1)e^{-x}\right]_0^1 + \int_0^1 2xe^{-x}\, dx = -2e^{-1} + 1 + 2\int_0^1 xe^{-x}\, dx$. Next let

$U = x$, $dV = e^{-x}\, dx \Rightarrow dU = dx$, $V = -e^{-x}$. By (6) again,

$\int_0^1 xe^{-x}\, dx = \left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\, dx = -e^{-1} + \left[-e^{-x}\right]_0^1 = -e^{-1} - e^{-1} + 1 = -2e^{-1} + 1$. So

$\int_0^1 (x^2+1)e^{-x}\, dx = -2e^{-1} + 1 + 2(-2e^{-1} + 1) = -2e^{-1} + 1 - 4e^{-1} + 2 = -6e^{-1} + 3$.

22. Let $u = \tan^{-1} x$, $dv = x\, dx \Rightarrow du = dx/\left(1+x^2\right)$, $v = \frac{1}{2}x^2$.

Then $\int x\tan^{-1} x\, dx = \frac{1}{2}x^2\tan^{-1} x - \dfrac{1}{2}\int \dfrac{x^2}{1+x^2}\, dx$. To evaluate the last integral, use long division or observe

that $\int \dfrac{x^2}{1+x^2}\, dx = \int \dfrac{\left(1+x^2\right) - 1}{1+x^2}\, dx = \int 1\, dx - \int \dfrac{1}{1+x^2}\, dx = x - \tan^{-1} x + C_1$. So

$\int x\tan^{-1} x\, dx = \frac{1}{2}x^2\tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1} x + C_1\right) = \frac{1}{2}\left(x^2\tan^{-1} x + \tan^{-1} x - x\right) + C$.

28. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w\, dw$. Thus, $\int_1^4 e^{\sqrt{x}}\, dx = \int_1^2 e^w 2w\, dw$. Now use parts with $u = 2w$, $dv = e^w\, dw$, $du = 2\, dw$, $v = e^w$ to get $\int_1^2 e^w 2w\, dw = \left[2we^w\right]_1^2 - 2\int_1^2 e^w\, dw = 4e^2 - 2e - 2\left(e^2 - e\right) = 2e^2$.