1. $W = \int_a^b f(x)\,dx = \int_0^9 \frac{10}{(1+x)^2}\,dx = 10\int_1^{10} \frac{1}{u^2}\,du \quad [u = 1+x,\ du = dx]$

$= 10\left[-\frac{1}{u}\right]_1^{10} = 10(-\frac{1}{10} + 1) = 9$ ft-lb

4. $25 = f(x) = kx = k(0.1)$ [10 cm = 0.1 m], so $k = 250$ N/m and $f(x) = 250x$. Now 5 cm = 0.05 m, so $W = \int_0^{0.05} 250x\,dx = \left[125x^2\right]_0^{0.05} = 125(0.0025) = 0.3125 \approx 0.31$ J.

In Exercises 7–12, n is the number of subintervals of length Δx, and x_i^* is a sample point in the ith subinterval $[x_{i-1}, x_i]$.

7. The portion of the rope from x ft to $(x + \Delta x)$ ft below the top of the building weighs $\frac{1}{2}\,\Delta x$ lb and must be lifted x_i^* ft, so its contribution to the total work is $\frac{1}{2}x_i^*\,\Delta x$ ft-lb. The total work is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \tfrac{1}{2}x_i^*\,\Delta x = \int_0^{50} \tfrac{1}{2}x\,dx = \left[\tfrac{1}{4}x^2\right]_0^{50} = \tfrac{2500}{4} = 625 \text{ ft-lb}$$

Notice that the exact height of the building does not matter (as long as it is more than 50 ft).

8. Each part of the top 10 ft of cable is lifted a distance x_i^* equal to its distance from the top. The cable weighs $\frac{60}{40} = 1.5$ lb/ft, so the work done on the ith subinterval is $\frac{3}{2}x_i^*\,\Delta x$. The remaining 30 ft of cable is lifted 10 ft. Thus,

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\tfrac{3}{2}x_i^*\,\Delta x + \tfrac{3}{2}\cdot 10\,\Delta x\right) = \int_0^{10} \tfrac{3}{2}x\,dx + \int_{10}^{40} \tfrac{3}{2}\cdot 10\,dx = \left[\tfrac{3}{4}x^2\right]_0^{10} + \left[15x\right]_{10}^{40}$$

$$= \tfrac{3}{4}(100) + 15(30) = 75 + 450 = 525 \text{ ft-lb}$$

10. The work needed to lift the bucket itself is 4 lb · 80 ft = 320 ft-lb. At time t (in seconds) the bucket is $x_i^* = 2t$ ft above its original 80 ft depth, but it now holds only $(40 - 0.2t)$ lb of water. In terms of distance, the bucket holds $\left[40 - 0.2(\frac{1}{2}x_i^*)\right]$ lb of water when it is x_i^* ft above its original 80 ft depth. Moving this amount of water a distance Δx requires $(40 - \frac{1}{10}x_i^*)\,\Delta x$ ft-lb of work. Thus, the work needed to lift the water is

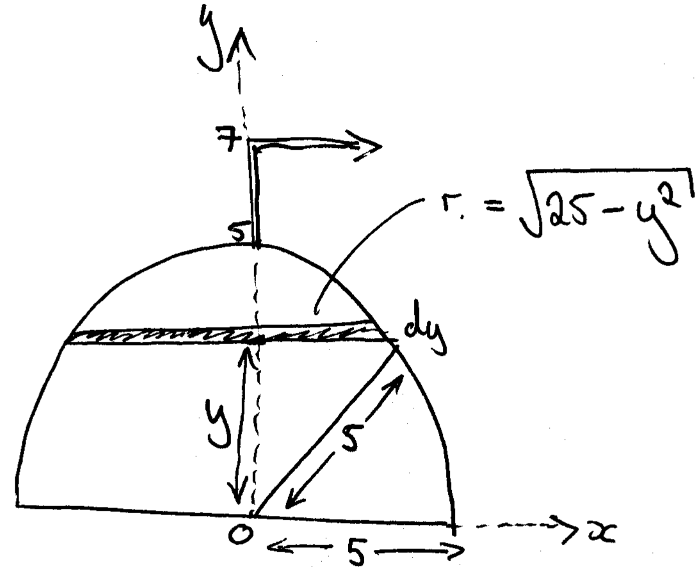
$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(40 - \tfrac{1}{10}x_i^*\right)\Delta x = \int_0^{80} \left(40 - \tfrac{1}{10}x\right)dx = \left[40x - \tfrac{1}{20}x^2\right]_0^{80} = (3200 - 320) \text{ ft-lb}$$

12. A horizontal cylindrical slice of water Δx ft thick has a volume of $\pi r^2 h = \pi \cdot 12^2 \cdot \Delta x$ ft^3 and weighs about $(62.5 \text{ lb/ft}^3)(144\pi\,\Delta x \text{ ft}^3) = 9000\pi\,\Delta x$ lb. If the slice lies x_i^* ft below the edge of the pool (where $1 \le x_i^* \le 5$) then the work needed to pump it out is about $9000\pi x_i^*\,\Delta x$. Thus,

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 9000\pi x_i^*\,\Delta x = \int_1^5 9000\pi x\,dx = \left[4500\pi x^2\right]_1^5 = 4500\pi(25 - 1) = 108{,}000\pi \text{ ft-lb}$$

Integration Handout A

#17)



- Mass of a disk in hemisphere $= \pi r^2 h \cdot \rho$
  $= \pi (25-y^2)\, dy\, 100$
- Force of a disk $= \pi(25-y^2) 100 \cdot 9.8 \cdot dy$
- Height disk moves to pump $= (7-y)$

$$\therefore W = \int_0^5 \pi(25-y^2)100 \cdot 9.8 \cdot (7-y)\, dy$$

$$= 980\pi \int_0^5 (175 - 25y - 7y^2 + y^3)\, dy$$

$$= 980\pi \left[175y - 12.5y^2 - \frac{7}{3}y^3 + \frac{y^4}{4}\right]_0^5$$

$$\boxed{= 1.315 \times 10^6 \text{ J}}$$