x+ K Y lem

6.1 #22 Problem Set 5

22. We know that the area under curve A between $t = 0$ and $t = x$ is $\int_0^x v_A(t)\,dt = s_A(x)$, where $v_A(t)$ is the velocity of car A and s_A is its displacement. Similarly, the area under curve B between $t = 0$ and $t = x$ is $\int_0^x v_B(t)\,dt = s_B(x)$.

(a) After one minute, the area under curve A is greater than the area under curve B. So A is ahead after one minute.

(b) The area of the shaded region has numerical value $s_A(1) - s_B(1)$, which is the distance by which A is ahead of B after 1 minute.

(c) After two minutes, car B is traveling faster than car A and has gained some ground, but the area under curve A from $t = 0$ to $t = 2$ is still greater than the corresponding area for curve B, so car A is still ahead.

(d) From the graph, it appears that the area between curves A and B for $0 \le t \le 1$ (when car A is going faster), which corresponds to the distance by which car A is ahead, seems to be about 3 squares. Therefore, the cars will be side by side at the time x where the area between the curves for $1 \le t \le x$ (when car B is going faster) is the same as the area for $0 \le t \le 1$. From the graph, it appears that this time is $x \approx 2.2$. So the cars are side by side when $t \approx 2.2$ minutes.

6.3 #1, 4

1. $y = 2 - 3x \quad \Rightarrow \quad L = \int_{-2}^{1} \sqrt{1 + (dy/dx)^2}\,dx = \int_{-2}^{1} \sqrt{1 + (-3)^2}\,dx = \sqrt{10}\,[1 - (-2)] = 3\sqrt{10}.$

The arc length can be calculated using the distance formula, since the curve is a line segment, so

$L = [\text{distance from } (-2, 8) \text{ to } (1, -1)] = \sqrt{[1 - (-2)]^2 + [(-1) - 8]^2} = \sqrt{90} = 3\sqrt{10}$

4. $y = 2^x \quad \Rightarrow \quad dy/dx = (2^x)\ln 2 \quad \Rightarrow \quad L = \int_0^3 \sqrt{1 + (\ln 2)^2\, 2^{2x}}\,dx$

6.5 #2, 11

2. $g_{ave} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{2}{9}\left[x^{3/2}\right]_1^4 = \frac{2}{9}(8 - 1) = \frac{14}{9}$

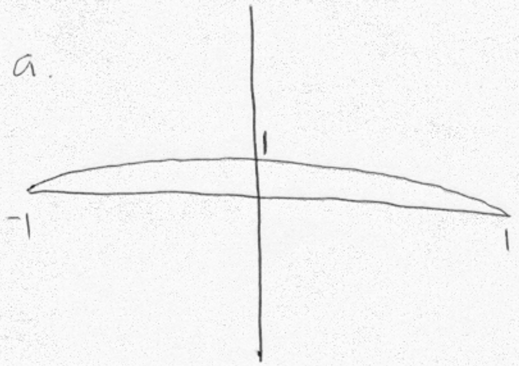
11. Let $t = 0$ and $t = 12$ correspond to 9 A.M. and 9 P.M., respectively.

$$T_{ave} = \frac{1}{12-0}\int_0^{12}\left[50 + 14\sin\tfrac{1}{12}\pi t\right]dt = \frac{1}{12}\left[50t - 14\cdot\tfrac{12}{\pi}\cos\tfrac{1}{12}\pi t\right]_0^{12}$$

$$= \frac{1}{12}\left[50\cdot 12 + 14\cdot\tfrac{12}{\pi} + 14\cdot\tfrac{12}{\pi}\right] = \left(50 + \tfrac{28}{\pi}\right)\,^\circ\text{F} \approx 59\,^\circ\text{F}$$

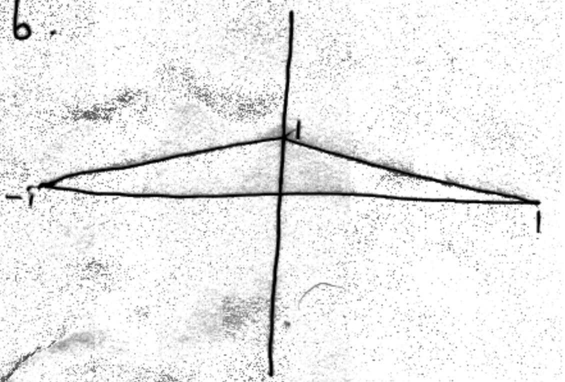
Integratio Handout A
Problem Set 5

16) a.



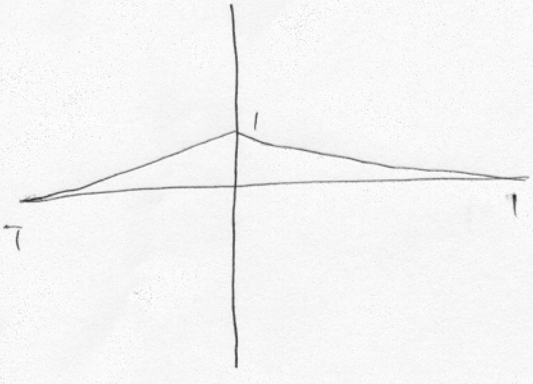
It appears that the function goes between values of 0 and 1, with the majority of points being closer to 1, so an approximate avg. value would be $\boxed{0.75}$

b.



Since both sides of the graph (i.e. on either side of the y-axis), they are increasing and decreasing with constant slope, differing only by a factor of -1. Thus a good guess would be $\boxed{0.5}$

c.



It seems that the average value would be higher than part (b) but not quite as high as part (a), so a good guess is $\boxed{0.65}$