

Assignment 30

Supplement

$$\textcircled{1} \quad y' - 4xy = x$$

$$y' = x(1 + 4y)$$

$$\frac{dy}{1+4y} = x dx$$

$$1+4y = A e^{2x^2}$$

$$y = A e^{2x^2} - \frac{1}{4}$$

$$\textcircled{2} \quad xy' + y = x^3$$

$$\frac{dy}{dx} + \frac{1}{x}y = x^2$$

$$y(x) = \frac{1}{x} \int x \cdot x^2 dx$$

$$y = \frac{1}{4} x^3 + \frac{c}{x}$$

$$\textcircled{3} \quad y' + y = e^x$$

$$y(x) = \frac{1}{e^x} \int e^x e^x dx$$

$$y(x) = \frac{e^x}{2} + \frac{c}{e^x}$$

$$y(0) = 6 \rightarrow c = 5\frac{1}{2}$$

$$y = \frac{e^x}{2} + \frac{11}{2e^x}$$

Handout A

$$\textcircled{10} \quad \frac{dy}{dt} = (y-3)^2$$

$$y = 3 - \frac{1}{t+c}$$

$$y(0) = 4 \rightarrow y = 3 - \frac{1}{t+5}$$

a) increasing

b) concavity changes

$$\textcircled{11} \quad \frac{dy}{dx} = y^2$$

$$y = \frac{-1}{x+c}$$

a) $y(0) = 0$

(i) y always zero

(ii) $y(0) = 0.01$
asymptote at $x = -100$
 y close to zero

(iii) $y(0) = -0.01$
asymptote at $x = 100$
 y close to zero

b) ~~unstable~~ semi-stable

$$c) \quad y = \frac{-1}{x-1}$$

d) $y \rightarrow \infty$

$$\textcircled{12} \quad a) \quad \frac{dw}{dt} = \frac{2}{5} - \frac{w(t)}{5}$$

$$2-w = A e^{-t/5}$$

$$w(0) = 0$$

$$w = -2 e^{-t/5} + 2$$

$$b) \quad \frac{db}{dt} = \frac{8}{5} - \frac{b(t)}{5}$$

$$b = A e^{-t/5} + 8$$

$$b = 2 e^{-t/5} + 8$$

$\textcircled{13}$ If this were true, y'' and y' must both be positive. However, from the equation we see that their sum is negative. Thus, y'' and y' cannot both be positive.