Homework 29

Supplement 31.3
1, 2, 3, 5, 6, 7
7.3 #33
7.4 #19

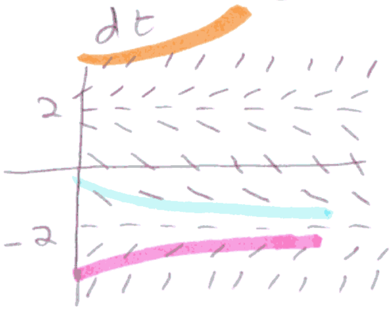
1. a) $\frac{dy}{dt} = 4y - 8$ $\qquad \frac{dy}{dt} = 0$ at $y = 2$

$y(0) = 0$

$y(0) = -1$

$y(0) = 3$

b) $\frac{dy}{dt} = y^2 - 4 = (y+2)(y-2)$ $\qquad \frac{dy}{dt} = 0$ at $y = \pm 2$



$y(0) = -1$

$y(0) = -3$

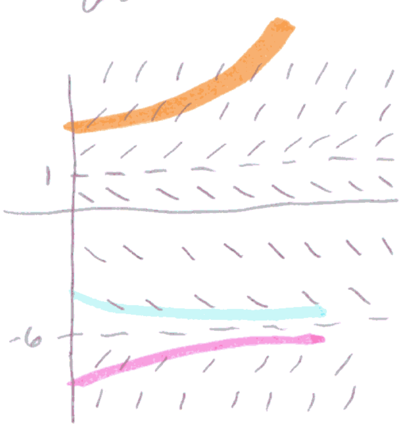
$y(0) = 4$

c) $\frac{dy}{dt} = (y-1)(y-2)(y+1)$

$y(0) = 0$

$y(0) = 3$

d) $\frac{dy}{dt} = y^2 + 5y - 6 = (y+6)(y-1)$



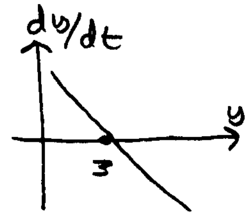
$y(0) = -5$

$y(0) = -7$

$y(0) = 2$

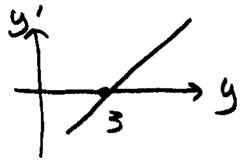
2. a) $\frac{dy}{dt} > 0$ for $y < 3$

$\frac{dy}{dt} < 0$ for $y > 3$

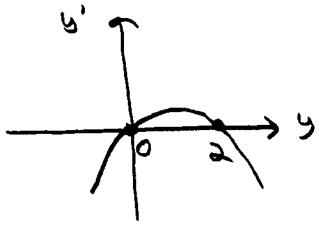


$$\frac{dy}{dt} = 3 - y$$

b) $\frac{dy}{dt} = y - 3$

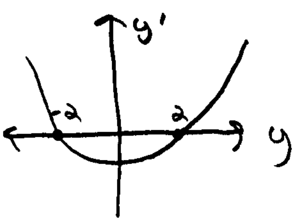


c)



$$\frac{dy}{dt} = -y(y-2)$$

d)



$$\frac{dy}{dt} = (y-2)(y+2)$$

3) a) $y=3$ is stable

b) $y=3$ is unstable

c) $y=2$ is stable

$y=0$ is unstable

d) $y=2$ is unstable

$y=-2$ is stable

5. a) $\frac{dP}{dt}$ must be positive or 0 for the town to survive. The critical point is $\frac{dP}{dt}=0$ (i.e. no population change)

$$\frac{dP}{dt} = .02P - N = 0$$

$N = .02P$ $\quad P = 100 \Rightarrow N = .02(100) = \boxed{2}$

b) $$\frac{dP}{dt} = .02P - 1000 = 0$$

$$.02P = 1000$$

$$P = 50{,}000 \text{ people}$$

6. $$\frac{dy}{dt} = y^2 - 1 = (y+1)(y-1)$$

Equilibrium points at $y=1$ and $y=-1$, so a, b, e are out. $\left.\frac{dy}{dt}\right|_{y=0} = -1 < 0$, so (d) is possible

7. $$\frac{dx}{dt} = x^2(2-x) \qquad x(0) = .5$$

$\left.\frac{dx}{dt}\right|_{x=.5} = (.5)^2(2-.5) = .375 > 0$, so the top two are out. Since $\left.\frac{dx}{dt}\right|_{x=0} = 0$, solutions cannot cross the x axis, so the lower left solution is possible.

33. (a) $\dfrac{dC}{dt} = r - kC \;\Rightarrow\; \dfrac{dC}{dt} = -(kC - r) \;\Rightarrow\; \displaystyle\int \frac{dC}{kC - r} = \int -dt \;\Rightarrow\; (1/k)\ln|kC - r| = -t + M_1$
$\Rightarrow\; \ln|kC - r| = -kt + M_2 \;\Rightarrow\; |kC - r| = e^{-kt+M_2} \;\Rightarrow\; kC - r = M_3e^{-kt} \;\Rightarrow$
$kC = M_3e^{-kt} + r \;\Rightarrow\; C(t) = M_4e^{-kt} + r/k$. $C(0) = C_0 \;\Rightarrow\; C_0 = M_4 + r/k \;\Rightarrow$
$M_4 = C_0 - r/k \;\Rightarrow\; C(t) = (C_0 - r/k)e^{-kt} + r/k$.

19. (a) $\dfrac{dP}{dt} = kP - m = k\left(P - \dfrac{m}{k}\right)$. Let $y = P - \dfrac{m}{k}$, so $\dfrac{dy}{dt} = \dfrac{dP}{dt}$ and the differential equation becomes
$\dfrac{dy}{dt} = ky$. The solution is $y = y_0e^{kt} \;\Rightarrow\; P - \dfrac{m}{k} = \left(P_0 - \dfrac{m}{k}\right)e^{kt} \;\Rightarrow\; P(t) = \dfrac{m}{k} + \left(P_0 - \dfrac{m}{k}\right)e^{kt}$.

(b) There will be an exponential expansion $\;\Leftrightarrow\; P_0 - \dfrac{m}{k} > 0 \;\Leftrightarrow\; m < kP_0$.

(c) The population will be constant if $P_0 - \dfrac{m}{k} = 0 \;\Leftrightarrow\; m = kP_0$. It will decline if $P_0 - \dfrac{m}{k} < 0 \;\Leftrightarrow$
$m > kP_0$.

(d) $P_0 = 8{,}000{,}000$, $k = \alpha - \beta = 0.016$, $m = 210{,}000 \;\Rightarrow\; m > kP_0$ $(= 128{,}000)$, so by part (c), the population was declining.