Differential Equation Handout A

① The rate of change of concentration inside the cell is $\frac{dC}{dt}$

Thus $\frac{dC}{dt} = K(L - C)$ where K is the proportionality constant.

Since the concentration of the solute in the cell is increasing over time K is positive.

②
a) $\frac{dI}{dt} = KI(N - I)$

Proportionality constant

Number of people who are not sick

b) If $\frac{dI}{dt}$ is positive, number of people who are sick is increasing.

If $\frac{dI}{dt}$ is negative, number of people who are sick is decreasing

Observe that $(N-I)\,I \geq 0$ always.

Thus if $K>0$ $\frac{dI}{dt}>0$ and thus the number of people who are infected is always increasing. (This is the most plausible scenario)

If $K<0$ $\frac{dI}{dt}<0$ and the number of people infected is always decreasing.

Given that $K>0$, people will stop getting infected only when $\frac{dI}{dt}=0$.

But this will only happen when $N=I$, which means that everyone is sick. Thus the model suggest that everyone will get infected at the end

③ a) $\frac{dM}{dt} = \text{rate }(M) = .04M$

Initial condition: $t=0$ $M=2000$ $\frac{dM}{dt}=80$

b) Since money is also added to the account

in this case $\frac{dM}{dt} = rM + 1000 = .4M + 1000$

Initial condition $t=0$ $M=2000$ $\frac{dM}{dt}=1080$

(4) According to the logic of compound interest, Elmer's debt is changing with the rate:

$$\frac{dB(t)}{dt} = \underset{\substack{\text{interest} \\ \text{accumulated} \\ \text{at time } t.}}{B \cdot r} - \underbrace{12000}_{\text{amount paid}}$$

$$= \underline{0.0725 \cdot B - 12000}$$

3. $y' = y - 1$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, IV is the direction field for this equation. Note that for $y = 1$, $y' = 0$.

4. $y' = y - x = 0$ on the line $y = x$, when $x = 0$ the slope is y, and when $y = 0$ the slope is $-x$. Direction field II satisfies these conditions. [Looking at the slope at the point $(0, 2)$, II looks more like it has a slope of 2 than does direction field I.]

5. $y' = y^2 - x^2 = 0 \quad \Rightarrow \quad y = \pm x$. There are horizontal tangents on these lines only in graph III, so this equation corresponds to direction field III.

6. $y' = y^3 - x^3 = 0$ on the line $y = x$, when $x = 0$ the slope is y^3, and when $y = 0$ the slope is $-x^3$. The graph is similar to the graph for Exercise 4, but the segments must get steeper very rapidly as they move away from the origin, because x and y are raised to the third power. This is the case in direction field I.