Problem Set #19

12. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$ satisfies (a) of the Alternating Series Test because $\frac{1}{(n+1)^4} < \frac{1}{n^4}$ and

(b) $\lim_{n\to\infty}\frac{1}{n^4} = 0$, so the series is convergent. Now $b_5 = 1/5^4 = 0.0016 > 0.001$ and

$b_6 = 1/6^4 \approx 0.00077 < 0.001$, so by the Alternating Series Estimation Theorem $n = 5$

13. Using the Ratio Test with the series $\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$,

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|a_{n+1}\cdot\frac{1}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{n+1}}{(n+1)!}\cdot\frac{n!}{(-2)^n}\right| = \lim_{n\to\infty}\left|\frac{-2}{n+1}\right|$$

$$= 2\lim_{n\to\infty}\frac{1}{n+1} = 2(0) = 0 < 1,$$

so the series is absolutely convergent (and therefore convergent). Now $b_7 = 2^7/7! \approx 0.025 > 0.01$ and $b_8 = 2^8/8! \approx 0.006 < 0.01$, so by the Alternating Series Estimation Theorem, $n = 7$. (That is, since the 8th term is less than the desired error, we need to add the first 7 terms to get the sum to the desired accuracy.)

19. Consider the series whose terms are the absolute values of the terms of the given series.
$\sum_{n=1}^{\infty}\left|\frac{(-1)^{n-1}}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$, which is a divergent p-series ($p = \frac{1}{2} \leq 1$). Thus, $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$ is *not* absolutely convergent.

13. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, so $a_n = \frac{x^n}{n!}$

For $x = 0.$, $|a_5|$ $8.3 \times 10^{-8} > 10^{8}$

but $|a_6| = 1.39 \times 10^{-9} < 10^{-8}$

So, $\sum_{0}^{5} \frac{(-0.1)^n}{n!}$ has error less than 10^{-8}

14. $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k)!}$, so $a_k = (-1)^k \frac{x^{2k+1}}{(2k)!}$

handout.

For $x = 0.2$, $|a_2| = 1.\overline{3} \times 10^{-5} > 10^{-6}$

but $|a_3| = 1.\overline{7} \times 10^{-8} < 10^{-6}$

So, $\sum_{k=0}^{2} (-1)^k \frac{(0.2)^{2k+1}}{(2k)!}$ has error less than 10^{-6}

The series is alternating, with the even terms positive, and the odd terms negative. So, approximations up to an even index are too large, and approximations up to an odd index are too small. Our approximation is up to an even index, so it is too large.

15. a) Divergent, by N^{th} term test: $|a_k| = \left|\frac{2k^2 - 10k}{10k^2 + 5k}\right| = \left|\frac{2k - 10}{10k + 5}\right|$

$\lim_{k \to \infty} |a_k| = \frac{2}{10} = \frac{1}{5} > 0$

Can't apply AST because terms don't tend to zero

b) Convergent, by absolute convergence: $|a_k| = \frac{|\sin k|}{\sqrt{k^3}} \leq \frac{1}{k^{3/2}}$, which

Can't apply AST because the series is not strictly alternating, since $\sin k$ can be positive or negative.