Assignment 17

Supplement:

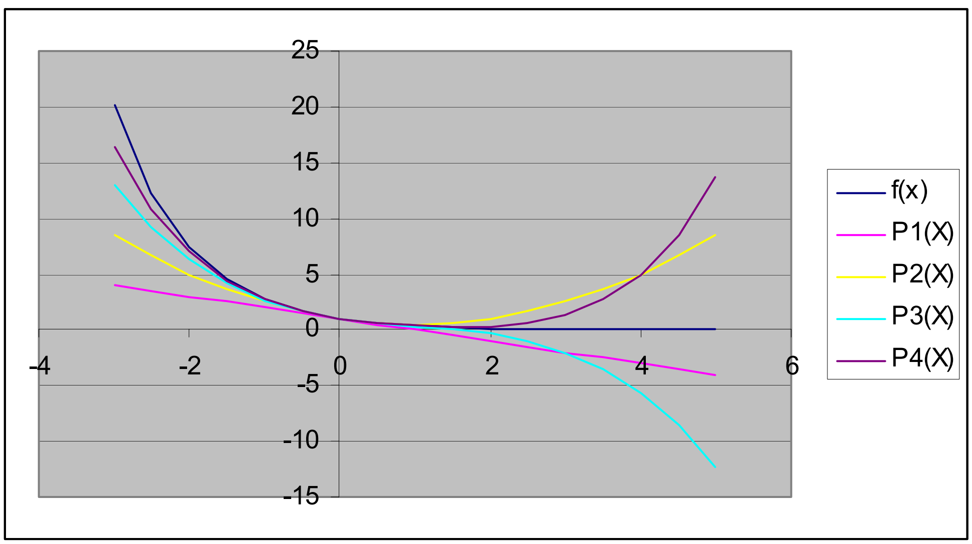
1)

a)

$$f(x) = e^{-x} \qquad f(0) = 1$$
$$f'(x) = -e^{-x} \qquad f'(0) = -1$$
$$f''(x) = e^{-x} \qquad f''(0) = 1$$
$$f'''(x) = -e^{-x} \qquad f'''(x) = -1$$
$$f^{(4)}(x) = e^{-x} \qquad f^{(4)}(x) = 1$$

$$= 1 - \frac{x}{1} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$$

b)



c)

x	e^{-x}	P1(x)	P2(x)	P3(x)	P4(x)
0.1	.904837	.9	.905	.904833	.904838
0.3	.740818	.7	.745	..7405	.740838

2.

a)

$$f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{x+1}$$

$$f''(x) = \frac{-1}{(x+1)^2}$$

$$f'''(x) = \frac{2}{(x+1)^3}$$

$$f^{(4)}(x) = \frac{-6}{(x+1)^4}$$

$$f(0) = 0$$

$$f'(0) = 1$$

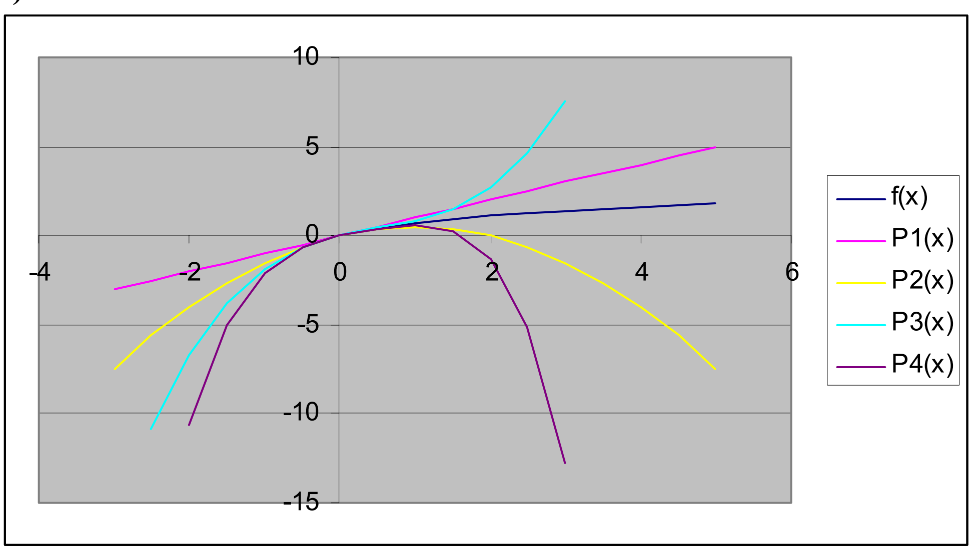
$$f''(0) = -1$$

$$f'''(0) = 2$$

$$f^{(4)}(0) = -6$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

b)



c)

x	e^{-x}	P1(x)	P2(x)	P3(x)	P4(x)
0.1	.095310	.1	.095	.09533	.0953083
0.3	.262364	.3	.255	.264	.2619750

9.

a_0 = negative because approximation is at f(0)
a_1 = positive because the first derivative is positive
a_2 = positive because 2^{nd} derivative shows concavity upward

11.

a)

$$f(x) = \tan(x)$$
$$f'(x) = \frac{1}{\cos^2 x}$$
$$f''(x) = \frac{2\sin(x)}{\cos^3 x}$$
$$f'''(x) = \frac{2\cos^2 x + 6\sin^2 x}{\cos^4 x}$$
$$f(0) = 0$$
$$f'(0) = 1$$
$$f''(0) = 0$$
$$f'''(0) = 2$$

$$= x + \frac{x^2}{2}$$

b) the x^2 term should be zero because sin is in the numerator, and the sin at 0 =0.

12.

a) a0 = negative (evaluate at f(0)); a1 = 0 (min of function); a2 = positive because concave upward (2^{nd} derivative test)
b) a0 = negative (same reason as in a); a1 = positive (slope positive); a2 = positive (2^{nd} derivative, concave up)
c) a0 = 0 (point of inflection); a1 = positive (slope upward); a2 = 0 (2^{nd} derivative at point of inflection)
d) a0 = positive; a1 = positive (slope positive); a2 = negative (concave downward at x=3)

13. $\sum_{k=1}^{n} (-1)^{k+1} \frac{x^k}{k}$

16. $\sqrt{103} \approx \sqrt{100} = 10$

$f(x) = \sqrt{x}$ $\qquad f(100) = 10$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad f'(100) = \frac{1}{20}$

$f''(x) = \frac{-1}{4x^{3/2}}$ $\qquad f''(100) = \frac{-1}{4000}$

$f(x) \approx 10 + \frac{1}{20}(x-100) - \frac{1}{8000}(x-100)^2$

Now Calculate:

$= 10 + \frac{1}{20} * 3 - \frac{1}{8000} * (3)^2 = 10.148875$

Real Answer: $\sqrt{103} = 10.14889$. Thus, this is a very close approximation using a Taylor expansion.

Series Handout A:

10.

$f(x) = x^3 - 4x^2 + 4x \Rightarrow f(0) = 0$

$f'(x) = 3x^2 - 8x + 4 \Rightarrow f'(0) = 4$

$f''(x) = 6x - 8 \Rightarrow f''(0) = -8$

$f^{(3)}(x) = 6 \Rightarrow f^{(3)}(0) = 6$

a) To get the best linear approximation, match the first derivative and y-intercept at x=0

$P_1(x) = 4x + 0 = 4x$

To get the best quadratic approximation, match the first and the second derivative and the y intercept at x=0

$P_2(x) = -4x^2 + 4x + 0 = (-4x^2) + 4x$

To get the best cubic approximation, match the first, the second, and the third derivative at x=0 and the y intercept:

$P_3 = x^3 - 4x^2 + 4x$

$\Rightarrow P_3(x) = f(x)$: this should not be a surprise; the function is a cubic, and the best cubic approximation to a cubic function is the function

b) Solve $f(x) = 0 \Rightarrow f(x) = x(x-2)^2 \Rightarrow x_1 = 0 \quad x = 2$

$f'(2) = 3(2^2) - 8(2) + 4 = 0 \Rightarrow x = 2$ is a local min (check with second derivative).

Since a_1 is the value of the first derivative and the value is 0, a_1 must be 0.

(Note: a_2, though, is not simply the value of the second derivative at $x=0$).

$a_4, a_5, \ldots a_n$ must be zero in our approximation of $f(x)$, for $f(x)$ is a cubic and can't be approximated by a higher degree polynomial: 4th and higher derivatives are all zeros, thus $a_4, \ldots, a_n$ must be zeros.

c) No matter what x we choose to approximate $f(x)$ around, 4th and higher derivatives of $f(x)$ will be zeros, thus $a_4, \ldots, a_n$ will be zeros.