Assignment 16

1) Graph of the picture shows a_n as the area of shaded boxes each under the curve, with width of 1 and height of a_n (thus the area is the value a_n). Therefore, $\sum_{n=2}^{\infty} \frac{1}{n^{1.3}}$ (i.e., the sum of the areas of all these boxes) $< \int_1^{\infty} \frac{1}{x^{1.3}} dx$. The integral converges with $p = 1.3 > 1$, so the series converges.

2) The figures for the two series are as follows: $\sum_{i=1}^{5} a_i$ is the area of the shaded boxes with width 1 and height $a_1, a_2, ..., a_5$. This spans the interval on the graph from $x = 1$ to $x = 6$. $\sum_{i=2}^{6} a_i$ is the set of shaded boxes with width 1 and height $a_2, a_3, ..., a_6$ - they all lie under the graph of $f(x)$. By looking at these graphs, we can see that the sum of areas of the first set of shaded boxes (and therefore the sum of the first 5 terms of a_i) is greater than the area under the curve (the integral of $f(x)$), while the areas of the second set of shaded boxes summates to a value less than the integral. Therefore, $\sum_{i=1}^{5} a_i > \int_1^6 f(x)dx > \sum_{i=2} 6a_i$.

5) $\sum_{n=1}^{\infty} n^b$ is a p-series with $p = -b$. $\sum_{n=1}^{\infty} b^n$ is a geometric series. B y statment 1 in §8.3, the p-series is convergent if $p > 1$. In this case, $p = -b$, so $-b > 1 \Rightarrow b < -1$ a re the values for which the series converge. A geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $||r| < 1$, so $\sum_{n=1}^{\infty} b^n$ converges if $|b| < 1 \Rightarrow -1 < b < 1$.

9) $\frac{1}{n^2+n+1} < \frac{1}{n^2}$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converg es by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges because it is a p-series with $p = 2 > 1$.

15) $f(x) = \dfrac{1}{x \ln x}$ is continuous and positive on $[2,\infty)$, and also decreasing since

$f'(x) = -\dfrac{1+\ln x}{x^2(\ln x)^2} < 0$ for x>2, so we can use the Integral Test.

$\int_2^{\infty} \dfrac{1}{x \ln x} dx = \lim_{t\to\infty}[\ln(\ln x)]_2^t = \lim_{t\to\infty}[\ln(\ln t) - \ln(\ln 2)] = \infty$, so the series diverges.

16) $\dfrac{2}{n^3+4} < \dfrac{2}{n^3}$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \dfrac{2}{n^3+4}$ converges by comparison with $\sum_{n=1}^{\infty} \dfrac{2}{n^3} = 2\sum_{n=1}^{\infty} \dfrac{1}{n^3}$, which converges because it is a constant multiple of a convergent p-series (p = 3 > 1).

18) $\dfrac{\sin^2 n}{n\sqrt{n}} \leq \dfrac{1}{n\sqrt{n}} = \dfrac{1}{n^{3/2}}$ and $\sum_{n=1}^{\infty} \dfrac{1}{n^{3/2}}$ converges (p = 3/2 > 1), so $\sum_{n=1}^{\infty} \dfrac{\sin^2 n}{n\sqrt{n}}$ converges by the Comparison Test.

22) Let $a_n = \dfrac{1}{n^3-n}$ and $b_n = \dfrac{1}{n^3}$. Then, $\sum_{n=2}^{\infty} a_n$ and $\sum_{n=2}^{\infty} b_n$ are series with positive terms and

$\lim_{n\to\infty} \dfrac{a_n}{b_n} = \lim_{n\to\infty} \dfrac{n^3}{n^3-n} = 1 > 0$. Since $\sum_{n=2}^{\infty} \dfrac{1}{n^3}$ is a convergent p-series without the n=1 term

(p = 3 > 1), $\sum_{n=2}^{\infty} \dfrac{1}{n^3-n}$ is convergent by the Limit Comparison Test.

Handout A

9)

a) $\dfrac{1}{n+2} > \dfrac{1}{n+n}$ for n > 2. $\dfrac{1}{n+n} = \dfrac{1}{2n} = (\dfrac{1}{2})*(\dfrac{1}{n})$, which means that this is really just ½ times the harmonic series, which diverges. Thus, this series must also diverge.

b) $\dfrac{1}{\sqrt{n^2+10}} > \dfrac{1}{\sqrt{n^2+n^2}}$ for n >3. Thus, $\dfrac{1}{\sqrt{n^2+n^2}} = \dfrac{1}{\sqrt{2}n} = (\dfrac{1}{\sqrt{2}})*(\dfrac{1}{n})$. This is just a constant times the harmonic series, which means that this series must also diverge.