

Assignment 15

Section 8.2

34) $\sum_{n=0}^{\infty} (2^n(x+1)^n) = \sum_{n=0}^{\infty} [2(x+1)]^n$.

$r = 2(x+1)$, so the series converges if $|r| < 1$:

$$|2(x+1)| < 1 \rightarrow |2x+2| < 1 \rightarrow -1 < 2x+2 < 1 \rightarrow -3 < 2x < -1 \rightarrow -\frac{3}{2} < x < -\frac{1}{2}.$$

The sum would be $\frac{1}{1-2(x+1)} = \frac{1}{-1-2x} = \frac{-1}{2x+1}$.

48) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \rightarrow \infty} a_n = 0$ by Theorem 6, so $\lim_{n \rightarrow \infty} \frac{1}{a_n}$ cannot be zero. Thus, $\sum_{n=1}^{\infty} \frac{1}{a_n}$ is divergent by the Test for Divergence.

51) The partial sums s_n form an increasing sequence, since $s_n - s_{n-1} = a_n > 0$ for all n . Also, the sequence s_n is bounded since $s_n \leq 1000$ for all n . So by the Monotonic Sequence Theorem, the sequence of partial sums converges and the series is convergent.

Extra credit # 52.

52) a) $\frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}} = \frac{f_n f_{n+1} - f_{n-1} f_n}{f_n^2 f_{n-1} f_{n+1}} = \frac{f_{n+1} - f_{n-1}}{f_n f_{n-1} f_{n+1}} = \frac{(f_{n-1} + f_n) - f_{n-1}}{f_n f_{n-1} f_{n+1}} = \frac{1}{f_{n-1} f_{n+1}}$

b) $\sum_{n=2}^{\infty} \left(\frac{1}{f_{n-1} f_n} \right) = \sum_{n=2}^{\infty} \left(\frac{1}{f_{n-1} f_n} - \frac{1}{f_n f_{n+1}} \right) = \lim_{n \rightarrow \infty} \left(\left(\frac{1}{f_1 f_2} - \frac{1}{f_2 f_3} \right) + \left(\frac{1}{f_2 f_3} - \frac{1}{f_3 f_4} \right) + \left(\frac{1}{f_3 f_4} - \frac{1}{f_4 f_5} \right) + \dots + \left(\frac{1}{f_{n-1} f_n} - \frac{1}{f_n f_{n+1}} \right) \right)$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{f_1 f_2} - \frac{1}{f_n f_{n+1}} \right) = \frac{1}{f_1 f_2} - 0 = \frac{1}{1 \cdot 1} = 1$$

c) $\sum_{n=2}^{\infty} \frac{f_n}{f_{n-1} f_{n+1}} = \sum_{n=2}^{\infty} \left(\frac{f_n}{f_{n-1} f_n} - \frac{f_n}{f_n f_{n+1}} \right) = \sum_{n=2}^{\infty} \left(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}} \right) = \lim_{n \rightarrow \infty} \left[\left(\frac{1}{f_1} - \frac{1}{f_3} \right) + \left(\frac{1}{f_2} - \frac{1}{f_4} \right) + \left(\frac{1}{f_3} - \frac{1}{f_5} \right) + \left(\frac{1}{f_4} - \frac{1}{f_6} \right) + \dots + \left(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}} \right) \right]$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{f_n} - \frac{1}{f_{n+1}} \right) = 1 + 1 - 0 - 0 = 2.$$

Section 8.3

3) a) We cannot say anything about $\sum a_n$. If $a_n > b_n$ for all n and $\sum b_n$ is convergent, then $\sum a_n$ could be convergent or divergent.

b) If $a_n < b_n$ for all n , then $\sum a_n$ is convergent.

4) a) If $a_n > b_n$ for all n , then $\sum a_n$ is divergent.

b) We cannot say anything about $\sum a_n$ for similar reasons as in question 3a.

10) $\frac{1}{2^{n-1}} > \frac{1}{2^n} = \frac{1}{2} * \frac{1}{2^n}$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{2^n}$, which diverges because it is a nonzero constant multiple of the divergent harmonic series.

19) $\frac{n+1}{n^2} > \frac{n}{n^2} = \frac{1}{n}$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$ diverges by comparison with the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

20) $\frac{4+3^n}{2^n} > \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$ diverges by comparison with the divergent geometric series $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$.