2. (a) Since $y = 1/(2x-1)$ is defined and continuous on $[1,2]$, the integral is proper.

(b) Since $y = \dfrac{1}{2x-1}$ has an infinite discontinuity at $x = \frac{1}{2}$, $\displaystyle\int_0^1 \frac{1}{2x-1}\,dx$ is a Type II improper integral.

(c) Since $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$ has an infinite interval of integration, it is an improper integral of Type I.

(d) Since $y = \ln(x-1)$ has an infinite discontinuity at $x = 1$, $\int_1^2 \ln(x-1)dx$ is a Type II improper integral.

6. $\displaystyle\int_2^{\infty} \frac{dx}{(x+3)^{3/2}} = \lim_{t\to\infty}\int_2^t \frac{dx}{(x+3)^{3/2}} = \lim_{t\to\infty}\left[\frac{-2}{\sqrt{x+3}}\right]_2^t = \lim_{t\to\infty}\left[\frac{-2}{\sqrt{t+3}} + \frac{2}{\sqrt{5}}\right] = \frac{2}{\sqrt{5}}$. Convergent

12. $I = \int_{-\infty}^{\infty}(2-v^4)\,dv = I_1 + I_2 = \int_{-\infty}^{0}(2-v^4)\,dv + \int_0^{\infty}(2-v^4)\,dv$, but
$I_1 = \lim_{t\to-\infty}\left[2v - \frac{1}{5}v^5\right]_t^0 = \lim_{t\to-\infty}\left(-2t + \frac{1}{5}t^5\right) = -\infty$. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

14. $\int_{-\infty}^{\infty} x^2e^{-x^3}\,dx = \int_{-\infty}^{0} x^2e^{-x^3}\,dx + \int_0^{\infty} x^2e^{-x^3}\,dx$, and
$\int_{-\infty}^{0} x^2e^{-x^3}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{3}e^{-x^3}\right]_t^0 = -\frac{1}{3} + \frac{1}{3}\left(\lim_{t\to-\infty} e^{-t^3}\right) = \infty$. Divergent

20. Since $f(r) = \dfrac{1}{r^2+4}$ is even,

$$I = \int_{-\infty}^{\infty} f(r)\,dr = 2\int_0^{\infty} f(r)\,dr = 2\lim_{t\to\infty}\int_0^t \frac{1}{r^2+4}\,dr = 2\lim_{t\to\infty}\left[\frac{1}{2}\arctan\frac{r}{2}\right]_0^t$$
$$= \lim_{t\to\infty}\left(\arctan\frac{t}{2} - 0\right) = \frac{\pi}{2}.\ \text{Convergent}$$

24. There is an infinite discontinuity at the left endpoint of $[0,3]$.
$$\int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t\to0^+}\int_t^3 \frac{dx}{x^{3/2}} = \lim_{t\to0^+}\left[\frac{-2}{\sqrt{x}}\right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t\to0^+}\frac{2}{\sqrt{t}} = \infty.\ \text{Divergent}$$

29. $\displaystyle\int_{-2}^{3}\frac{dx}{x^4} = \int_{-2}^{0}\frac{dx}{x^4} + \int_0^3\frac{dx}{x^4}$, but $\displaystyle\int_{-2}^{0}\frac{dx}{x^4} = \lim_{t\to0^-}\left[-\frac{x^{-3}}{3}\right]_{-2}^t = \lim_{t\to0^-}\left[-\frac{1}{3t^3} - \frac{1}{24}\right] = \infty$. Divergent

51. (a) $I = \int_{-\infty}^{\infty} x\,dx = \int_{-\infty}^{0} x\,dx + \int_0^{\infty} x\,dx$, and
$\int_0^{\infty} x\,dx = \lim_{t\to\infty}\int_0^t x\,dx = \lim_{t\to\infty}\left[\frac{1}{2}x^2\right]_0^t = \lim_{t\to\infty}\left[\frac{1}{2}t^2 - 0\right] = \infty$, so I is divergent.

(b) $\int_{-t}^{t} x\,dx = \left[\frac{1}{2}x^2\right]_{-t}^{t} = \frac{1}{2}t^2 - \frac{1}{2}t^2 = 0$, so $\lim_{t\to\infty}\int_{-t}^{t} x\,dx = 0$. Therefore, $\int_{-\infty}^{\infty} x\,dx \neq \lim_{t\to\infty}\int_{-t}^{t} x\,dx$.