

Mathematics 1b - Solution Set for PS 1

1 Textbook problems

$$5.2.18 \int_1^5 \frac{e^x}{1+x} dx$$

$$5.2.20 \int_1^4 \sqrt{x} dx$$

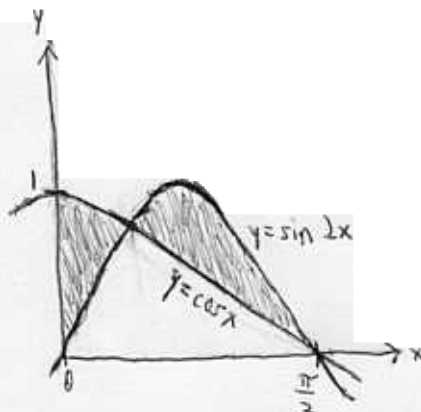
5.2.32 A look at the graph of this function reveals that the sought-for area is the same as the area of a semicircle with radius 2, i.e., $\int_{-2}^2 \sqrt{4-x^2} dx = \frac{1}{2} \cdot \pi \cdot 2^2 = 2\pi$.

5.2.35 Again looking at the graph of the function, it is clear that each of the 2 smaller triangles below the x -axis have areas of $\frac{1}{2}$ the area of the large triangle above the x -axis. Thus, everything cancels out, and $\int_{-2}^2 1 - |x| dx = 0$

6.1.2 The shaded area is equivalent to the area bounded above by $y = \sqrt{x+2}$ and $y = \frac{1}{x+1}$ on the interval $[0, 2]$. Thus, we can use an integral:

$$\int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1} \right) dx = \left. \frac{2}{3}(x+2)^{3/2} - \ln|x+1| \right|_0^2 = \frac{16 - 4\sqrt{2}}{3} - \ln 3$$

6.1.25 The graph:



$y = \cos x$ is on top until the intersection point, then $y = \sin 2x$ is on top. The intersection point can be found by simultaneously solving the equations (remembering that our solution should be in $[0, \frac{\pi}{2}]$):

$$\begin{aligned} \sin 2x &= \cos x \\ 2 \sin x \cos x &= \cos x \\ \sin x &= \frac{1}{2} \\ x &= \frac{\pi}{6} \end{aligned}$$

Thus, the area between the curves is:

$$\begin{aligned} \int_0^{\pi/6} (\cos x - \sin 2x) dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x) dx &= \left(\sin x + \frac{1}{2} \cos 2x \right) \Big|_0^{\pi/6} + \left(-\frac{1}{2} \cos 2x - \sin x \right) \Big|_{\pi/6}^{\pi/2} \\ &= \left(\frac{3}{4} - \frac{1}{2} \right) + \left(-\frac{1}{2} - \left(-\frac{3}{4} \right) \right) \\ &= \frac{1}{2} \end{aligned}$$

2 Handout A

Since we are told which function bounds the area above and which function bounds it below, all that really needs to be done is the calculation of the bounds. This is done by solving the equations simultaneously.

$$\begin{aligned} x &= -x^2 + 2 \\ x^2 + x - 2 &= 0 \\ (x + 2)(x - 1) &= 0 \end{aligned}$$

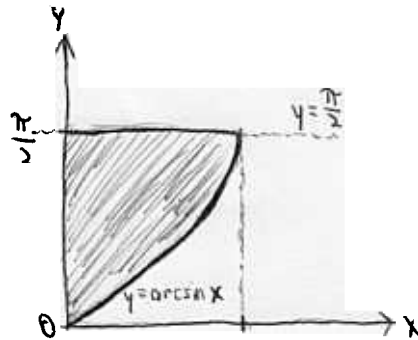
This implies $x = -2$ or $x = 1$. Thus, our integral will be over the interval $[-2, 1]$

$$\int_{-2}^1 (-x^2 + 2 - x) dx$$

2. The easiest way to do this is to integrate with respect to y .

$$\begin{aligned}\arcsin x &= y \\ x &= \sin y\end{aligned}$$

We should make sure to adjust our bounds accordingly; a quick glance at the graph shows us what we need:



Thus, the area will be:

$$\int_0^{\pi/2} \sin y \, dy = -\cos y \Big|_0^{\pi/2} = 1$$