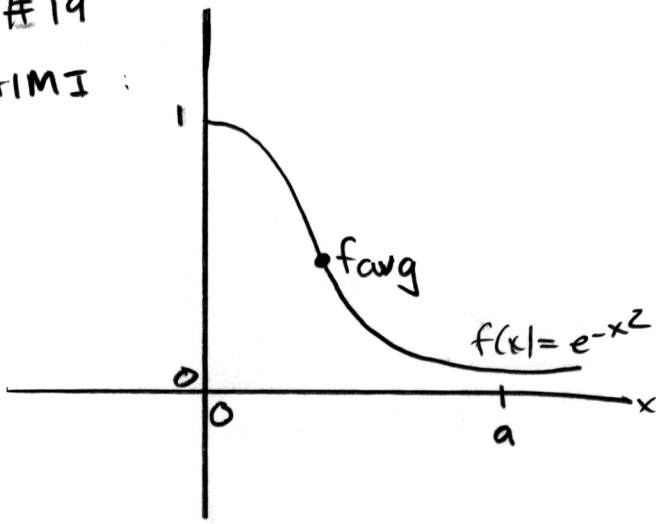


HO #19

CLAIM I:



$$0 < \int_0^a e^{-x^2} dx < a$$

$$0 < \frac{1}{a} \int_0^a e^{-x^2} dx < 1$$

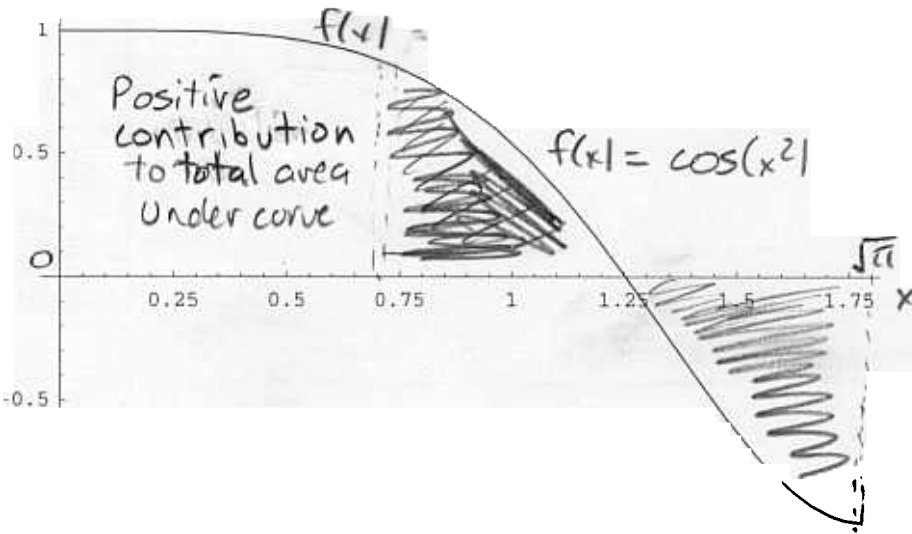
} These are equivalent statements.

f_{avg}

By definition, the average value of the function must lie somewhere between $f(x)=0$ and $f(x)=1$

TRUE

CLAIM II:

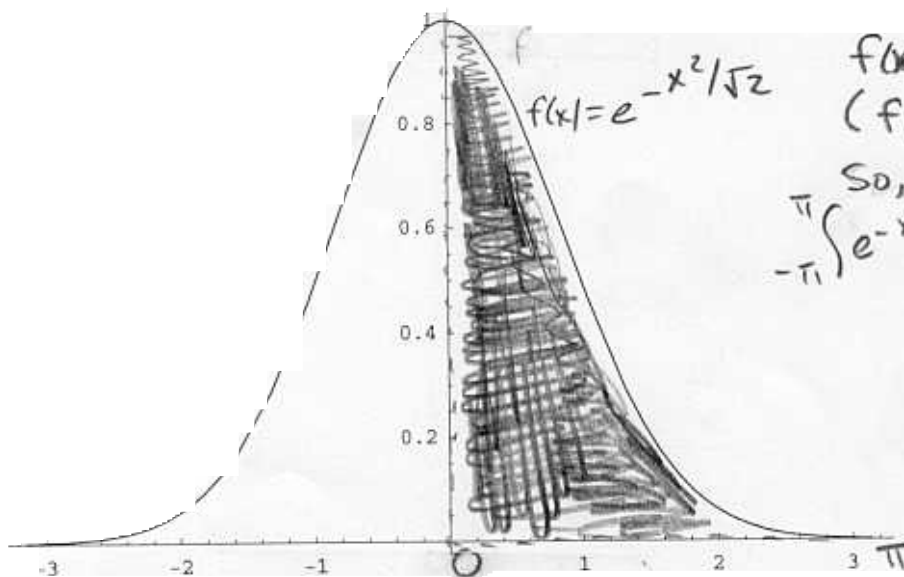


By inspection, we can see that the shaded areas are equal

$$\int_0^{\sqrt{\pi}} \cos(x^2) dx > 0$$

TRUE

CLAIM III



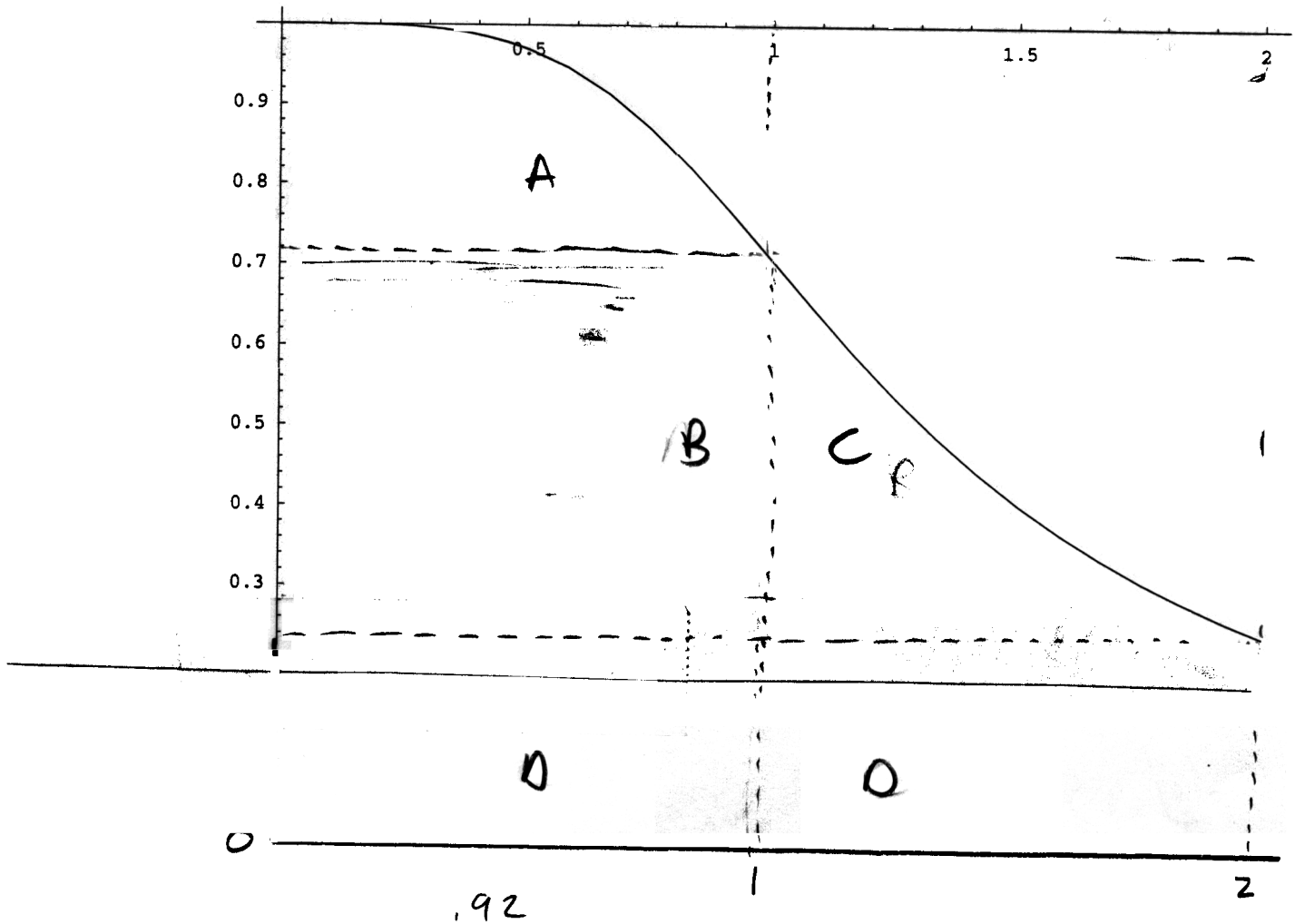
$f(x)$ is even, by inspection ($f(-x) = f(x)$).

So,

$$\int_{-\pi}^{\pi} e^{-x^2/\sqrt{2}} dx = 2 \int_0^{\pi} e^{-x^2/\sqrt{2}} dx$$

TRUE

CLAIM 4



$$\int_0^1 \frac{1}{\sqrt{1+x^4}} dx = \text{Area}_A + \text{Area}_B + \text{Area}_D$$

$$\int_1^2 \frac{1}{\sqrt{1+x^4}} dx = \text{Area}_C + \text{Area}_D$$

By inspection, $\text{Area}_B > \text{Area}_C$.

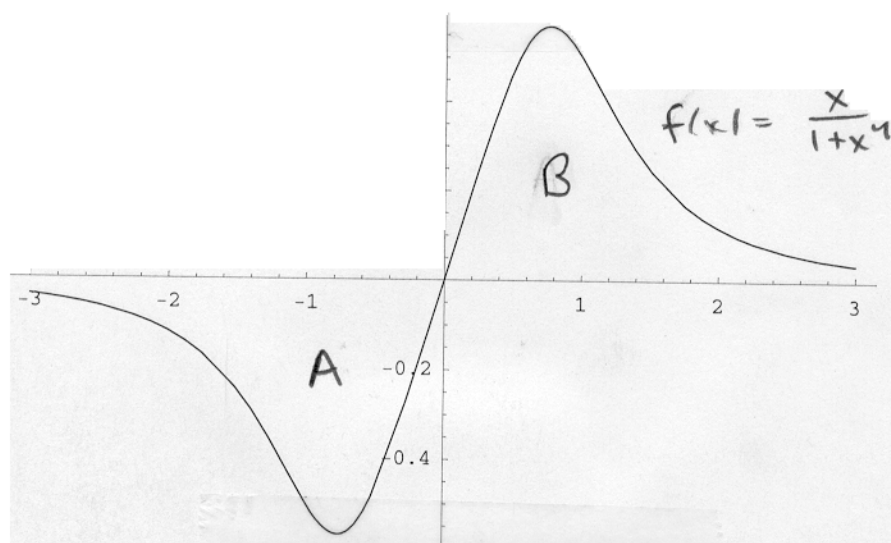
Compare $\text{Area}_A + \text{Area}_B$ and Area_C as Area_D are the same.

We know $\text{Area}_B > \text{Area}_C$ so $\text{Area}_A + \text{Area}_B$ must also be greater than Area_C (as $\text{Area}_A, B, C > 0$)

So, $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx > \int_1^2 \frac{1}{\sqrt{1+x^4}} dx$

The claim was false

CLAIM 5



$f(x)$ appears to be odd, by inspection.

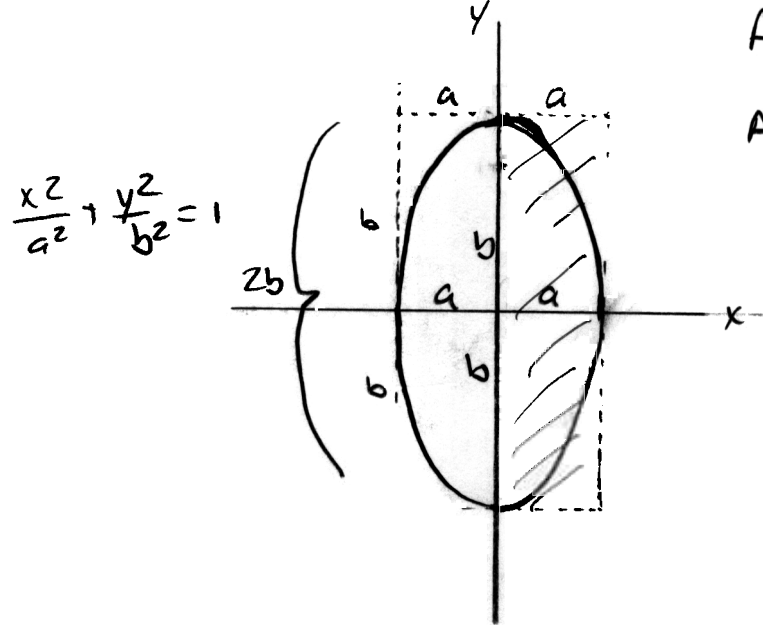
$$[f(-x) = -f(x)]$$

Both regions A and B appear to have the same area.

$$\therefore \int_0^3 \frac{x}{1+x^4} dx = - \int_{-3}^0 \frac{x}{1+x^4} dx \Rightarrow \int_{-3}^3 \frac{x}{1+x^4} dx = 0 < .001$$

CLAIM IS FALSE.

CLAIM 6



Area = $2ab$

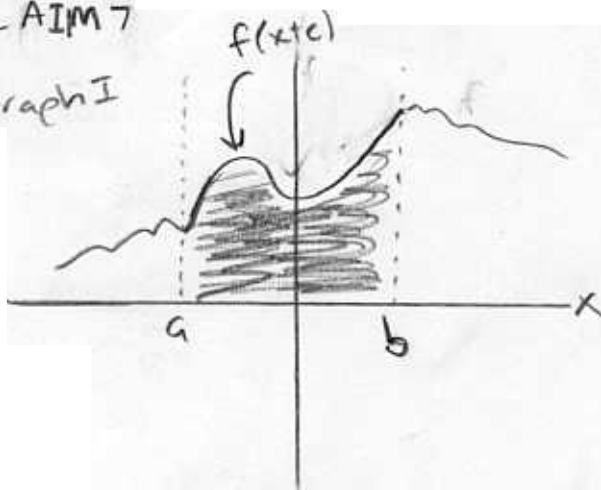
Area = $2(2ab) = 4ab$

By inspection, we find the area of the ellipse, A , can be expressed in the following relation;

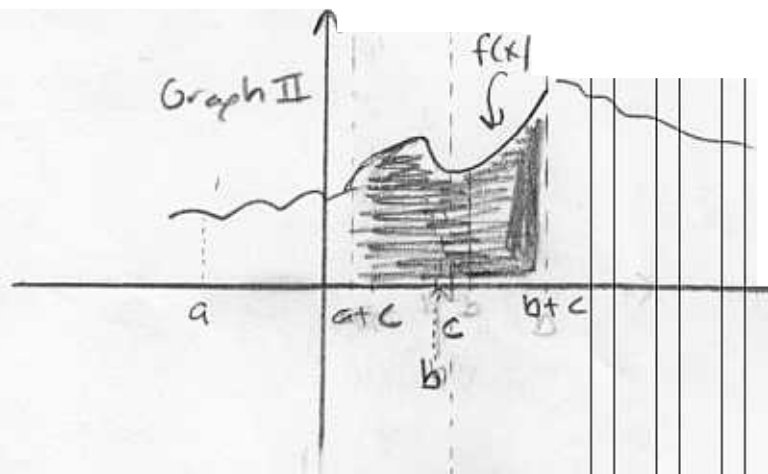
$$2ab < A < 4ab. \quad \text{TRUE.}$$

CLAIM 7

Graph I



Graph II



I have chosen an arbitrary function $f(x)$ on an x -interval $[a, b]$.

The shaded area in Graph I is $\int_a^b f(x+c) dx$.

The shaded area in Graph II is $\int_{a+c}^{b+c} f(x) dx$.

In Graph I, the function $f(x)$ has been shifted to the left c units. (Or, equivalently, the function $f(x+c)$ has been shifted to the right c units. This case is drawn above.)

We can see by inspection that we are finding the area under essentially two identical curves

With the fundamental theorem of calculus, we would observe,
the same result,

Given $F = f'$

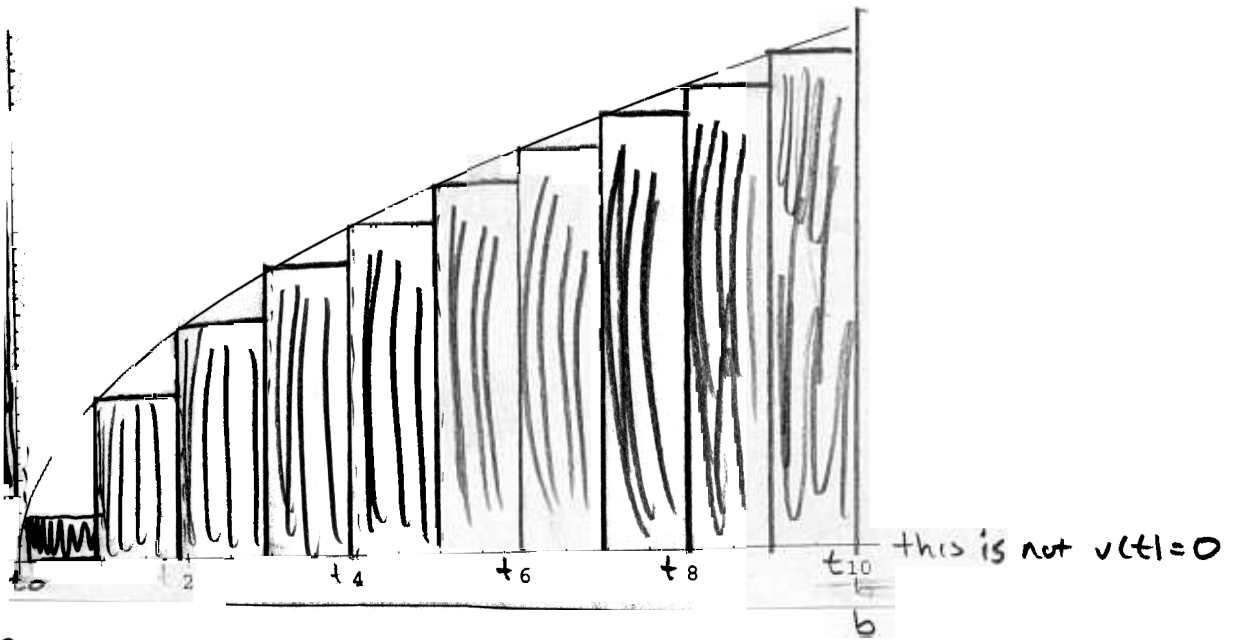
$$\int_a^b f(x+c) dx = F(x+c) \Big|_a^b = F(b+c) - F(a+c).$$

$$\int_{a+c}^{b+c} f(x) dx = F(x) \Big|_{a+c}^{b+c} = F(b+c) - F(a+c),$$

TRUE.

CLAIM 8

Area under $v(t) = \text{total distance}$



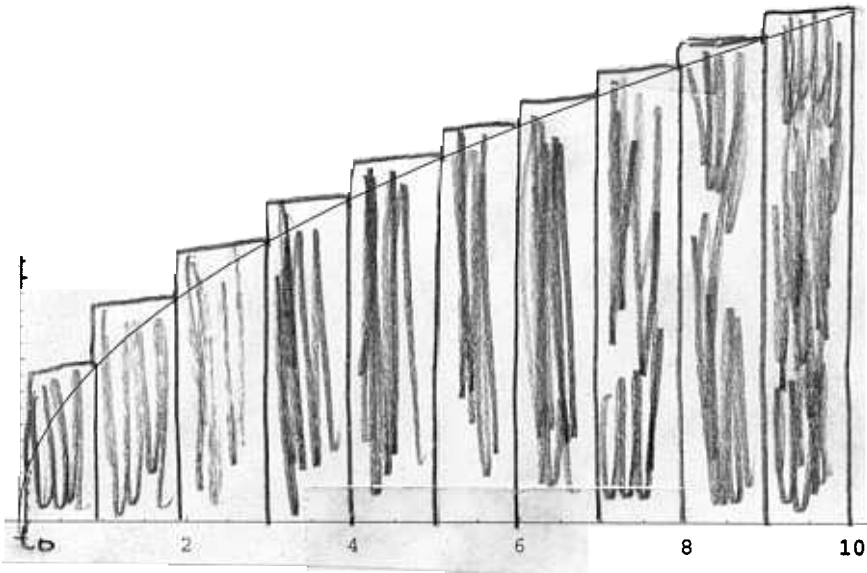
$\sum_{k=1}^{10} v(t_{k-1}) \Delta t = \text{Approximation of Area under graph using left-hand endpoints.}$

\therefore This underestimates the area under the curve.

$$\sum_{k=1}^{10} v(t_{k-1}) \Delta t < \text{distance traveled on } [a, b]$$

CLAIM IS FALSE

CLAIM 9



$\sum_{k=1}^{10} v(t_{k-1}) \Delta t = \text{Approximation of area under graph using left-hand endpoints}$

\therefore This overestimates the area under the curve.

$$\sum_{k=1}^{10} v(t_k) \Delta t > \text{distance traveled on } [a, b], \quad \text{CLAIM IS TRUE}$$