

Solutions to Integration Problems

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1. Perform integration by parts with $u = x, dv = \sec^2 x dx$. The answer is $x \tan x + \ln |\cos x| + C$
2. Use the fact that $4x + 4 = 2(2x + 1) + 2$. The answer is $2x + \ln |2x + 1| + C$
3. Perform the substitution $u = x^2$. The answer is $\frac{7}{2}e^{x^2} + C$
4. Use the fact that $x^2 + 6x + 9 = (x + 3)^2$. The answer is $-\frac{1}{x+3} + C$
5. Since $\sin(x^3)$ is an odd function and the interval is symmetric around 0, the integral is zero. (Note that we would not want to find an antiderivative for this function.) 0
6. Perform the substitution $u = 3x + 4$. The answer is $\frac{2}{9}(31\sqrt{31} - 7\sqrt{7})$
7. Perform the substitution $u = e^x$ followed by $u = \sin \theta$. The answer is $\frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}e^x\sqrt{1 - e^{2x}} + C$
8. Integrate by parts twice and note that we obtain the original integrand. The answer is $\frac{1}{2}e^x(\sin x - \cos x) + C$
9. Integrate by parts with $u = \ln x, dv = x dx$. The answer is $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$
10. Use the trigonometric identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$. The answer is $\frac{1}{2}x - \frac{1}{28}\sin(14x) + C$
11. Perform the substitution $u = x^4 + x^3 + x^2 + x + 1$. The answer is $\ln |x^4 + x^3 + x^2 + x + 1| + C$
12. Use partial fractions. The answer is $\frac{9}{2}\ln|x - 1| - 15\ln|x - 2| - \frac{21}{2}\ln|x - 3| + C$
13. $\frac{1}{2}\sin(x^2) + C$. Substitute for x^2 .
14. $x^2 \sin x + 2x \cos x - 2 \sin x + C$. Integrate by parts twice.
15. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(4n+1) \cdot (2n)!}$.
16. $\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$. Use trig identity $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$
17. $\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2}\cos(x^2) + C$. Substitute for x^2 .
18. 1. Integrate by parts.
19. $\sin(x) - \frac{1}{3}\sin^3(x) + C$. Use trig identity $\cos^2 x = 1 - \sin^2 x$. Substitute for $\sin^2 x$.
20. $x(\ln x)^2 - 2x \ln x + 2x + C$. Integrate by parts twice.
21. $\frac{1}{3}(\ln x)^3 + C$. Substitute for $\ln x$
22. $\ln 3$. As above.
23. $\frac{1}{2}(x^2 + 1)\ln(x^2 + 1) - \frac{1}{2}(x^2 + 1) + C$. Substitute for x^2 and then do by parts. Your answer may differ by a constant, of course.
24. $x - 2 \arctan x + C$. Realize that the fraction equals $1 - \frac{2}{x^2+1}$.
25. $x - \ln|x + 1| + \ln|x - 1| + C$. Note $1 + \frac{2}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$
26. $\frac{1}{2}\tan^{-1}(x^2) + C$.
27. $\sqrt{x^2 + 1} + C$. Substitute for x^2 .
28. $\frac{1}{4}\ln|e^x + 2| - \frac{1}{4}\ln|e^x - 2| + C$. Substitute for e^x followed by partial fractions
29. $\frac{1}{2}\ln|e^{2x} - 4| + C$. Substitute for e^{2x} .
30. $2\sqrt{7} - 2\sqrt{5}$. Substitute for $e^x + 4$.
31. $2\ln|x - 3| - \ln|x + 1| + C$. Partial fractions.
32. $x + 2 \tan^{-1} x + C$. Realize that the fraction equals $1 + \frac{2}{x^2+1}$.
33. $\frac{1}{3}e^{x^3} + C$. Substitute for x^3 .
34. $-\frac{3}{2}\ln|\cos 2x| + C$. Write as $\frac{\sin x}{\cos x}$ and then substitute for $\cos x$.
35. $e^{\tan x} + C$. Substitute for $\tan x$.
36. $-\frac{\cos(2 \sin x)}{2} + C$. Substitute for $\sin x$.
37. $\frac{\ln(1+x)}{(1+x)} + \frac{1}{(1+x)} + C$. Integrate by parts.
38. $-\frac{2}{1+x} + C$. Factor the denominator and look at the problem.
39. $\frac{1}{2}\ln \frac{x}{x+2} + C$. Partial Fractions.

40. $\frac{x}{9\sqrt{1-9x^2}} - \frac{1}{27} \sin^{-1}(3x) + C$. Integrate by parts. Look to complete the integration by noticing the exact integral you are evaluating appearing in the integration by parts. Bring it over to the other side of the equation.
41. $3x \sin x + 3 \cos x + C$. Integrate by parts.
42. $\frac{1}{3}(x^2 + 2x)^{\frac{3}{2}} + C$. Write as $(x+1)\sqrt{x^2+2x}$. Substitute for x^2+2x .
43. $1 - \cos(1)$. Substitute for $\ln x$.
44. $\frac{x}{2}(\sin(\ln(x)) - \cos(\ln(x))) + C$. Integrate by parts twice, look to switch terms over.
45. $\frac{-1}{3x} - \frac{\arctan(x/3)}{9}$. Partial Fractions.
46. $\frac{x^2}{2} - \frac{1}{2} \ln(x^2+1) + C$. Write fraction as $\frac{x(x^2+1)-x}{x^2+1}$. Split the fraction and split the integral across the minus sign. Substitute for x^2 on the right side.
47. $\ln|x| - \frac{1}{2} \ln|x^2+1| + C$. Partial Fractions.
48. $\frac{1}{3} \ln \left| \frac{1-\cos(x)}{4-\cos(x)} \right| + C$. Substitute for $\cos x$. Partial fractions.
49. $2e^{\sqrt{x}}$. Substitute for \sqrt{x} .
50. $\frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4} \sin^{-1}(\frac{2x}{3}) + C$. Look at no. 18.
51. $\frac{1}{2} \ln(x^2+1) + \tan^{-1}(x)$. Write the fraction as $\frac{x}{x^2+1} + \frac{1}{x^2+1}$.
52. $e^{(x+1)^2}$. Write as $(2x+2)e^{(x+1)^2}$. Substitute for $(x+1)^2$.
53. $\frac{\sin(2x)}{2}$. Use Trig. Identity $\cos(2x) = \cos^2(x) - \sin^2(x)$.
54. $-\sqrt{1-x^2}$. Substitute for x^2 .
55. $-\frac{1}{2} \cos(x^2-1)$. Substitute for x^2-1 .
56. $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$. Factor $x^5-1 = (x-1)(x^4+x^3+x^2+x+1)$
57. $\frac{(x^2+1)^5}{5} + \frac{2(x^2+1)^3}{3} + x^2 + 1$. Substitute $u = x^2+1$.
58. 0. Realize $\frac{x^3}{\sqrt{1-x^2}}$ is an odd function.
59. $2e^3/9 + 1/9$. Integrate by parts
60. $-\frac{1}{4} \cos(2x) + C$. Substitute $u = \cos(x)$.
61. $\ln \ln \ln x + C$. Substitute $u = \ln(x)$.
62. $x(1-x^2)^{-1/2} + C$. Substitute $x = \sin(u)$.
63. $-\frac{1}{2} \cos 2x - \frac{1}{6} \cos^3 2x + C$ Substitute $u = 2x$. Remember $\sin^2(u) = 1 - \cos^2(u)$.
64. $x \ln \sqrt{x} - x/2 + C$ Remember $\ln(x^a) = a \ln(x)$. Integrate by parts.
65. $\frac{2}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| + C$. Partial Fractions.
66. $\frac{x}{\sqrt{1+x^2}}$. Substitute $x = \tan(u)$.
67. $e^{e^x} + C$. Substitute $u = e^x$.
68. $\ln(1 + \sin(x))$. Substitute $u = 1 + \sin(x)$.
69. $\frac{x^2}{2} - \ln|x-1|$. Factor numerator. Then use $u = x-1$.
70. $\frac{\ln|x|}{a^2} - \frac{\ln|x^2+a^2|}{2a^2}$. Partial Fractions.
71. $\lim_{t \rightarrow 1} -\frac{\sqrt{1-x^2}}{x} \Big|_{1/2}^t = \sqrt{3}$. Substitute $x = \sin(u)$.
72. $\lim_{t \rightarrow \infty} -\frac{1+\ln(x)}{x} \Big|_1^t = 1$. Integrate by parts.
73. $\frac{1}{2} \ln \left(\frac{2-\sqrt{2}}{3(\sqrt{2}+2)} \right)$. Substitute $u = x^2 - a^2$.
74. $\frac{x^2(\ln x)^2}{2} - \frac{x^2}{2} \ln x + \frac{x^2}{4} + C$. Integrate by parts twice.
75. $3(\ln 2 - \frac{1}{2})$. Substitute $u = x^{\frac{1}{3}}$.
76. $\tan x - x + C$. Use Trig. Identity $\tan^2(u) = \sec^2(u) - 1$.
77. $\frac{\pi}{12}$. Substitute $u = e^x$.
78. $\frac{\pi}{2}$. Substitute $x = \sin(u)$.
79. $-\frac{1}{36}$. Integrate by parts.
80. $\frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{e}} \right)$. Substitute $u = -\frac{1}{x}$.
81. $\sqrt{x^2+3} + C$. Substitute $u = x^2+3$.
82. $x^2/2 + C$. Realize $e^{\ln x} = x$.
83. $1/2 \ln(x^2+1) - \ln|x+1| + C$. Partial Fractions.
84. $\pi/16$. Substitute $x = \sin u$.
85. $t \ln t - t + C$, where $t = \sin x$. Substitute $u = \sin x$. Then integrate by parts.
86. $2(t - \arctan(t)) + C$, where $t = \sqrt{x}$. Substitute $u^2 = x$.
87. $(2/3)t^{3/2} + C$, where $t = \ln x$. Substitute $u^2 = \ln x$.
88. $(x^2 - 2x + 2)e^x + C$. Integrate by parts twice.
89. $(\cos x)^3/3 - \cos x + C$. Use $\sin^2(u) = 1 - \cos^2(u)$.
90. $x \ln(x^2+1) - 2x + 2 \arctan(x) + C$. Integrate by parts.
91. $t \ln t - t + C$, where $t = \ln x$. Substitute $u = \ln x$. Then by parts.