Solution set #23

Math 1b

Problems: 6.5 4, 7, 10, 12, 19, 22

Extra: 17, 18

4) $25 = f(x) = kx = k(.1)$ [10cm = .1m]

$k = 250\ N/m \quad f(x) = 250x$

$W = \int_0^{.05} 250x\,dx = [125x^2]_0^{.05} = (125)(.0025) = \boxed{.31 J}$

7) $W = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2} x_i^* \Delta x = \int_0^{50} \frac{1}{2} x\,dx = [\frac{1}{4}x^2]_0^{50} = \frac{2500}{4} = \boxed{625\ ft\cdot lb}$

10) Bucket: 4lb · 80ft = 320 ft-lb $\quad x_i^* = 2t$ ft $\quad$ with $(40-.2t)$lb H_2O

$W = \lim_{n\to\infty} \sum_{i=1}^{n} (40 - \frac{1}{10}x_i^*)\Delta x = \int_0^{80} (40 - \frac{1}{10}x)\,dx = [40x - \frac{1}{20}x^2]_0^{80} = 3200 - 320$

320 for bucket $= \boxed{3200\ ft\text{-}lb}$

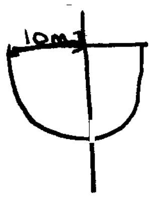
12) 

slice: $V = \pi r^2 h = \pi\, 12^2 \cdot \Delta x\ ft^3$

$(62.5\ lb/ft^3)(144\pi\,\Delta x\, ft) = 9000\pi\,\Delta x\, lb$

$W = \lim_{n\to\infty} \sum_{i=1}^{n} 9000\pi\, x_i^*\,\Delta x = \int_1^5 9000\pi x\,dx = [4500\pi x^2]_1^5 = 4500\pi(25-1)$

$\boxed{108{,}000\pi\ ft\text{-}lb}$

19) 

$x^2 + y^2 = 10^2 \quad y = \sqrt{100 - x^2} \quad$ width $= 2y = 2\sqrt{100 - (x_i^*)^2}$

Area: $2\sqrt{100 - (x_i^*)^2}\,\Delta x \quad$ pressure: $\rho g x_i^* = \rho g x_i^*\, 2\sqrt{100 - (x_i^*)^2}\,\Delta x$

$F = \lim_{n\to\infty} \sum_{i=1}^{n} \rho g x_i^*\, 2\sqrt{100 - (x_i^*)^2}\,\Delta x$

$\int_0^{10} \rho g x \cdot 2\sqrt{100 - x^2}\,dx = 9.8\times10^3 \int_0^{10} \sqrt{100 - x^2}\,dx$

$9.8\times10^3 \int_{100}^{0} u^{1/2}(-du) \quad [u = 100 - x^2;\ du = -2x\,dx]$

$9.8\times10^3 \int_0^{100} u^{1/2}\,du = 9.8\times10^3 [\frac{2}{3}u^{3/2}]_0^{100} =$

$\boxed{6.5\times10^6\ N}$

22) Area: $2\sqrt{2y_i^*}\,\Delta y$ pressure $\delta d_i = \delta(8-y_i^*)$

$$F = \int_0^8 \delta(8-y)\,2\sqrt{2y}\,dy = 42\times 2\cdot\sqrt{2}\int_0^8 (8-y)\,y^{1/2}\,dy$$

$$= 84\sqrt{2}\int_0^8 (8y^{1/2} - y^{3/2})\,dy = 84\sqrt{2}\left[8\cdot \tfrac{2}{3} y^{3/2} - \tfrac{2}{5} y^{5/2}\right]_0^8$$

$$= \boxed{5734.4\text{ lb}}$$

Extra Credit

17) a) $F = G\frac{m_1 m_2}{r^2}$ $\quad W = \int_a^b F(r)\,dr = \int_a^b G\frac{m_1 m_2}{r^2}\,dr = Gm_1m_2\left[\frac{-1}{r}\right]_a^b =$

$Gm_1m_2\left(\frac{1}{a} - \frac{1}{b}\right)$

b) $W = GMm\left(\frac{1}{R} - \frac{1}{R+1{,}000{,}000}\right)$ $\quad M = M_E \quad R = R_E \quad m = $ mass of satellite

$W = (6.67\times10^{-11})(5.98\times10^{24})(1000)\times\left(\frac{1}{6.37\times10^6} - \frac{1}{7.37\times10^6}\right) = \boxed{8.50\times10^9\text{ J}}$

18) a) $W = \int_R^\infty \frac{GMm}{r^2}\,dr = \lim_{t\to\infty}\int_R^t \frac{GMm}{r^2}\,dr = \lim_{t\to\infty} GMm\left[\frac{-1}{r}\right]_R^t =$

$GMm \lim_{t\to\infty}\left(-\frac{1}{t} + \frac{1}{R}\right) = \frac{GMm}{R}$

$$W = \frac{6.67\times10^{-11}\times 5.98\times10^{24}\cdot 10^3}{6.37\times10^6} = \boxed{6.26\times10^{10}\text{ J}}$$

b) $W = \frac{GMm}{R}$ initial kinetic energy supplies work: conservation of E

$\frac{1}{2}mv_0^2 = \frac{GMm}{R} \Rightarrow \boxed{v_0 = \sqrt{\frac{2GM}{R}}}$