0.1 Problem Set 6

Read §8.6 Do: §8.5, #20abc, 22, 23ab, 28

20) Through the power series theorem (thm 3) and the problem's given convergences, we find that the Radius of Convergence is $R = 4$ ($-4 < x < 4$ converges).
a) $\sum_{n=0}^{\infty} c_n$ shows that $x = 1$. It's inside the Radius of Convergence, so it converges.
b) $\sum_{n=0}^{\infty} c_n 8^n$ shows that $x = 8$, so it diverges by the same reasoning.
c) $\sum_{n=0}^{\infty} c_n(-3)^n$ shows that $x = -3$, so it converges.

22) The partial sums cannot converge on $(1, \infty)$, because $f(x)$ is negative, and all the partial sums are positive. It cannot converge on $(-\infty, -1)$, because $f(x)$ stays between 0 and 1, and all partial sums are either greater or less than that interval. Therefore (as supported by these eliminations and the graph), the partial sums seem to converge on $(-1, 1)$.

23)a) If $a_n = \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$, then the Ratio Test gives us $\lim_{n\to\infty} |\frac{x^{2n+3}}{(n+1)!(n+2)!2^{2n+3}} \cdot \frac{n!(n+1)!2^{2n+1}}{x^{2n+1}}| = (\frac{x}{2})^2 \lim_{n\to\infty} \frac{1}{(n+1)(n+2)} =$ 0 for all x. So $J_1(x)$ converges for all x and it's domain is $(-\infty, \infty)$.

28) $\sum c_n x^{2n}$ can be written as $\sum c_n(x^2)^n$. Therefore, the series converges wherever $|x^2| < R \Rightarrow |x| < \sqrt{R}$. So the Radius of Convergence is $\sqrt{R}$.

Do: §8.6 #6, 12, 16, 28

6) $f(x) = \frac{1}{1+9x^2} = \frac{1}{1-(-9x^2)} = \sum_{n=0}^{\infty}(-9x^2)^n = \sum_{n=0}^{\infty}(-1)^n 3^{2n} x^{2n}$. The series converges when $|-9x^2| < 1$, or $|X| < \frac{1}{3}$. $I = (-\frac{1}{3}, \frac{1}{3})$.

12)a) Let $g(x) = \frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$. (Geometric Series with $R = 1$.)

$$f(x) = ln(1+x) = \int \frac{dx}{1+x} = \int [\sum_{n=0}^{\infty}(-1)^n x^n]dx = C + \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n} \tag{1}$$

$C = 0$, because $f(0) = 0$. $R = 1$.
b) $f(x) = (x)ln(1+x) = x[\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}]$ (part (a)), $= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{n+1}}{n} = \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n-1}$ where $R = 1$.

16) From Example 7, $g(x) = arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$. So,

$$f(x) = arctan(\frac{x}{3}) = \sum_{n=0}^{\infty}(-1)^n \frac{(\frac{1}{3})^{2n+1}}{2n+1} = \sum_{n=0}^{\infty}(-1)^n \frac{1}{3^{2n+1}(2n+1)} x^{2n+1} \tag{2}$$

So $|\frac{X}{3}| < 1 \Rightarrow |x| < 3 \Rightarrow R = 3$.

28) $\int_0^{0.5} \frac{dx}{1+x^6}$ is an integral they don't know how to do yet, plus the problem says to use a power series. So,

$$\int_0^{0.5} \sum_{n=0}^{\infty}(-1)^n x^{6n} dx = \sum_{n=0}^{\infty}[\frac{(-1)^n x^{6n+1}}{6n+1}]_0^{0.5} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(6n+1)2^{6n+1}} = \frac{1}{2} - \frac{1}{7 \cdot 2^7} + \frac{1}{13 \cdot 2^{13}} - \frac{1}{19 \cdot 2^{19}} \cdots \tag{3}$$

Since this is an alternating series, remember the error theorem. Since $\frac{1}{19 \cdot 2^{19}} \approx 1.0 \cdot 10^{-7}$, to six decimal places, $\int_0^{0.5} \frac{dx}{1+x^6} \approx \frac{1}{2} - \frac{1}{7 \cdot 2^7} + \frac{1}{13 \cdot 2^{13}} \approx 0.498893$.

Plus:

1) A power series $\sum_{n=0}^{\infty} c_n(x-1)^n$ has a radius of convergence $R = 5$. So $|x-1| < 5 \Rightarrow -5 < x-1 < 5 \Rightarrow -4 < x < 6$. The four possibilities for the interval of convergence are $(-4, 6), [-4, 6), (-4, 6], [-4, 6]$.

2) The interval of convergence for $\sum_{n=0}^{\infty} c_n(x-a)^n$ is $(-3, 1]$.
a) So we work backwards. $-3 < x < 1 \Rightarrow -2 < x+1 < 2$, so $a = -1$ (and $R = 2$).

b) Since we are given that the interval of convergence is $(-3, 1]$, $x = -2.8$ converges (it is within the interval), while $x = 2.8$ does not.