Problem Set #32

1. (a) $dx/dt = -0.05x + 0.0001xy$. If $y = 0$, we have $dx/dt = -0.05x$, which indicates that in the absence of y, x declines at a rate proportional to itself. So x represents the predator population and y represents the prey population. The growth of the prey population, $0.1y$ (from $dy/dt = 0.1y - 0.005xy$), is restricted only by encounters with predators (the term $-0.005xy$). The predator population increases only through the term $0.0001xy$; that is, by encounters with the prey and not through additional food sources.

   (b) $dy/dt = -0.015y + 0.00008xy$. If $x = 0$, we have $dy/dt = -0.015y$, which indicates that in the absence of x, y would decline at a rate proportional to itself. So y represents the predator population and x represents the prey population. The growth of the prey population, $0.2x$ (from $dx/dt = 0.2x - 0.0002x^2 - 0.006xy = 0.2x(1 - 0.001x) - 0.006xy$), is restricted by a carrying capacity of 1000 [from the term $1 - 0.001x = 1 - x/1000$] and by encounters with predators (the term $-0.006xy$). The predator population increases only through the term $0.00008xy$; that is, by encounters with the prey and not through additional food sources.

2. (a) $dx/dt = 0.12x - 0.0006x^2 + 0.00001xy$. $dy/dt = 0.08y + 0.00004xy$.
   The xy terms represent encounters between the two species x and y. An increase in y makes dx/dt (the growth rate of x) larger due to the positive term $0.00001xy$. An increase in x makes dy/dt (the growth rate of y) larger due to the positive term $0.00004xy$. Hence, the system describes a cooperation model.

   (b) $dx/dt = 0.15x - 0.0002x^2 - 0.0006xy = 0.15x(1 - x/750) - 0.0006xy$.
   $dy/dt = 0.2y - 0.00008y^2 - 0.0002xy = 0.2y(1 - y/2500) - 0.0002xy$.
   The system shows that x and y have carrying capacities of 750 and 2500. An increase in x reduces the growth rate of y due to the negative term $-0.0002xy$. An increase in y reduces the growth rate of x due to the negative term $-0.0006xy$. Hence, the system describes a competition model.

Handout 6

4.)

a) Competitive

b) $\frac{dx}{dt} = 0.1x - 0.05xy = 0$

$\Rightarrow x = 0$ or $y = 2$

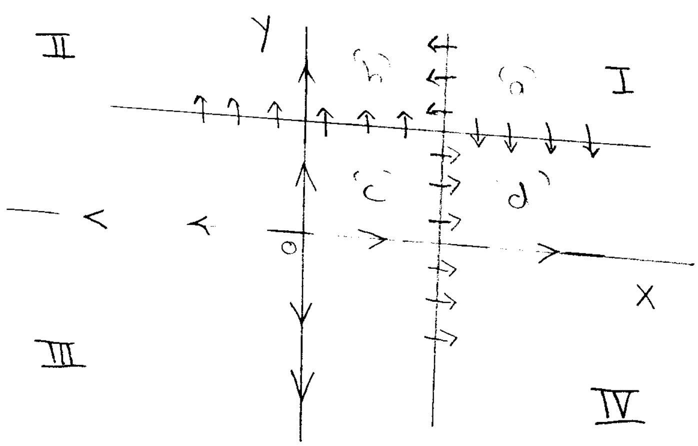
$\frac{dy}{dt} = 0.1y - 0.05xy = 0$

$\Rightarrow y = 0$ or $x = 2$

$\therefore$ Equilibrium population: $x = 2$
$y = 2$

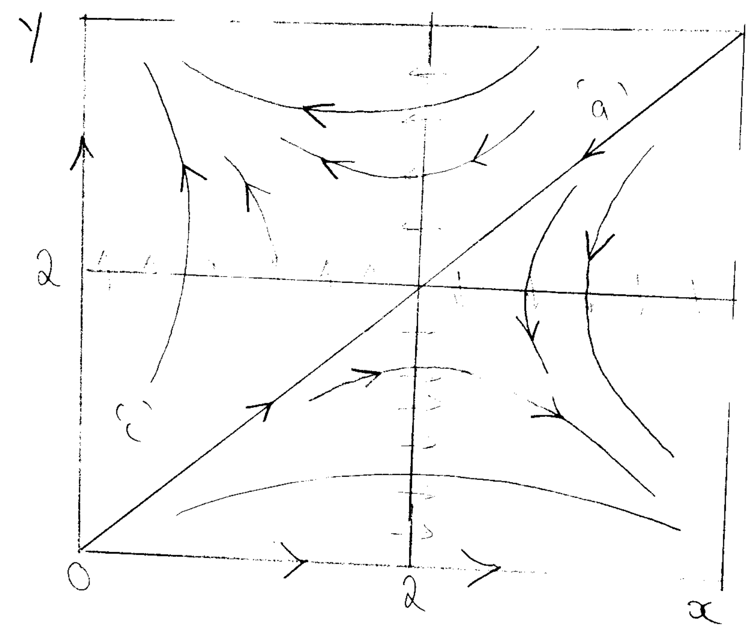
c) Nullclines at $\frac{dx}{dt} = 0$ or $\frac{dy}{dt} = 0$

$\Rightarrow$ Nullclines at $x = 0, y = 2, y = 0, x = 2$



Direction given by $\frac{dy}{dt}$ and $\frac{dx}{dt}$ at the values of the nullclines.

d) for regions 'a', 'b', 'c' & d



e) If $x = 0$, y increases indefinitely (see nullcline)
If $y = 0$, x increases indefinitely (see nullcline)

f) see 'd'.

g) i) Population of x will increase, population of y will decrease. Eventually x will dominate, y will be extinct.
ii) Same as 'i' with x and y switched.
iii) Same as 'i'.

h) Support? (not entirely sure what Darwin's argument was though).

Handout H

1.) $$\frac{dx}{dt} = ax - bx^2 - cxy = x(a - bx) - cxy$$

$$\frac{dy}{dt} = -dy + exy$$

a) x – prey (increases with $y = 0$)
y – predator (decreases with $x = 0$).

b) • expect $\frac{a}{b}$ quantity of prey (carrying capacity)
• expect all predators to die (no food).

c) Equilibrium at $\frac{dx}{dt} = \frac{dy}{dt} = 0$

$$\Rightarrow \frac{dx}{dt} = x\left[a - bx - cy\right] = 0$$

$\therefore$ either $x = 0$ or $y = \frac{a - bx}{c} = \frac{a}{c} - \frac{bx}{c}$

and $$\frac{dy}{dt} = y\left[-d + ex\right] = 0$$

$\therefore$ either $y = 0$ or $x = \frac{d}{e}$ plugging in

$\Rightarrow$ Equilibrium population at $\left(\frac{d}{e}, \frac{a}{c} - \frac{b}{c}\cdot\frac{d}{e}\right)$

Q.E.D

d) explanation of the constants.

2.) $\frac{dx}{dt} = x(1-x) - axy$

$\frac{dy}{dt} = y(1-y) - axy$

a) No rats: expect to see 1 mouse (carrying capacity)

No mice: expect to see 1 rat

b) for $a = \frac{1}{2}$ nullclines at $x = 0$, $y = 2(1-x) \Rightarrow y = 2 - 2x$

$y = 0$, $x = 2(1-y) \Rightarrow y = 1 - \frac{1}{2}x$