

Mathematics 1b - Solution Set for PS 26

Section 7.1 #9, 11, 12, 14

9. (a) $\frac{dP}{dt} = 1.2P(1 - P/4200)$. Now $\frac{dP}{dt} > 0 \Rightarrow 1 - P/4200 > 0$ [assuming that $P > 0$] $\Rightarrow P/4200 < 1 \Rightarrow P < 4200 \Rightarrow$ the population is increasing for $0 < P < 4200$.

(b) $\frac{dP}{dt} < 0 \Rightarrow P > 4200$

(c) $\frac{dP}{dt} = 0 \Rightarrow P = 4200$ or $P = 0$

11. First graph: This function is increasing *and* also decreasing. But $dy/dt = e^t(y-1)^2 \geq 0$ for all t , implying that the graph of the solution of the differential equation cannot be decreasing on any interval.

Second graph: When $y = 1$, $dy/dt = 0$, but the graph does not have a horizontal tangent line.

12. The correct equation is C.

A is not correct, because $y' = 1 + xy > 1$ for points in the first quadrant, but from the graph we can see that $y' < 0$ for some points in the first quadrant.

B is not correct, because $y' = -2xy = 0$ when $x = 0$, but we can see that $y' > 0$ for $x = 0$.

So C is correct by elimination, and because it equals 1 when $x = 0$, and is greater than 1 in the second quadrant, and is less than one in the first quadrant.

14. (a) The coffee cools most quickly as soon as it is removed from the heat source. The rate of cooling decreases toward 0 since the coffee approaches room temperature.

(b) $\frac{dy}{dt} = k(y - R)$, where k is a proportionality constant, y is the temperature of the coffee, and R is the room temperature. The initial condition is $y(0) = 95^\circ C$. The answer and the model support each other because as y approaches R , dy/dt approaches 0, so the model seems appropriate.

(c) See the graph on page 521 in the solutions manual. It is a decreasing graph with y-intercept at 95 and a horizontal asymptote at 20.

Section 7.2 #1

1.(a) See graph on page 522 of the solutions manual.

(b) It appears that the constant functions $y = 0$, $y = -2$, and $y = 2$ are equilibrium solutions. Note that these three values of y satisfy the given differential equation $y' = y(1 - (1/4)y^2)$.

Problems 2 and 3 from Handout F

2. The graph of $f(x)$ can never be both increasing and concave up because, if $f(x)$ is increasing, then $y' > 0$. Since $y' + y'' = -x^2 \Rightarrow y' = -x^2 - y''$, then $-x^2 - y'' > 0 \Rightarrow y'' < -x^2$ if $f(x)$ is increasing. Since $-x^2$ is always less than or equal to zero, $y'' < 0$ if $f(x)$ is increasing, so it is concave down.

3. (a) Note that if $f''(x) = 0$, $f(x)$ is a linear function $Ax + B$.

$$\begin{aligned} f_{ave} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{b-a} [A \frac{x^2}{2} + Bx]_a^b = \frac{1}{b-a} [\frac{Ab^2}{2} + Bb - \frac{Aa^2}{2} - Ba], \\ &= \frac{1}{b-a} [\frac{A(b^2 - a^2)}{2} + B(b-a)] = A(\frac{a+b}{2}) + B = f(\frac{a+b}{2}), \end{aligned}$$

which is the value of the function at the midpoint of the interval.

(b) If $f''(x) > 0$, then the rate at which $f(x)$ changes is getting faster. Hence, points with greater y value will lie farther on the x axis, and the average will be skewed to the right, making it greater than the midpoint.

(c) The same idea as (b), only in the other direction.