Mathematics 1b - Solution Set for PS 25

Section 7.1 # 1, 3, 4, 6

1.$y = x - x^{-1} \Rightarrow y' = 1 + x^{-2}$. To show that y is a solution of the differential equation, we will substitute the expressions for y and y' in the left-hand side of the equation and show that the left-hand side is equal to the right-hand side.

$$LHS = xy' + y = x(1 + x^{-2}) + (x - x^{-1}) = x + x^{-1} + x - x^{-1} = 2x = RHS$$

3. (a) $y = \sin kt \Rightarrow y' = k \cos kt \Rightarrow y'' = -k^2 \sin kt$
$y'' + 9y = 0 \Rightarrow -k^2 \sin kt + 9 \sin kt = 0 \Rightarrow (9 - k^2) \sin kt = 0[\text{for all } t] \Rightarrow 9 - k^2 = 0 \Rightarrow k = \pm 3$
(b) $y = A \sin kt + B \cos kt \Rightarrow y' = Ak \cos kt - Bk \sin kt \Rightarrow y'' = -Ak^2 \sin kt - Bk^2 \cos kt$
$y'' + 9y = 0 \Rightarrow -Ak^2 \sin kt - Bk^2 \cos kt + 9(A \sin kt + B \cos kt) = 0 \Rightarrow (9 - k^2)[A \sin kt + B \cos kt] = 0$
The final equation is true for all values of A and B if $k = \pm 3$

4. $y = e^{rt} \Rightarrow y' = re^{rt} \Rightarrow y'' = r^2 e^{rt}$.
$y'' + y' - 6y = 0 \Rightarrow r^2 e^{rt} + re^{rt} - 6e^{rt} = 0 \Rightarrow e^{rt}(r^2 + r - 6) = 0$.
e^{rt} is never zero, so $(r^2 + r - 6) = 0 \Rightarrow (r + 3)(r - 2) = 0 \Rightarrow r = -3 \text{or} 2$

6. (a)$y = Ce^{x^2/2} \Rightarrow y' = Ce^{x^2/2}(2x/2) = xCe^{x^2/2} = xy$
(b) See graph p. 520 of solution manual.
(c) $y(0) = 5 \Rightarrow Ce^0 = 5 \Rightarrow C = 5$, so the solution is $y = 5e^{x^2/2}$
(d) $y(1) = 2 \Rightarrow Ce^{1/2} = 2 \Rightarrow C = 2e^{-1/2}$, so the solution is $y = 2e^{-1/2}e^{x^2/2} = 2e^{(x^2-1)/2}$

Handout p.990 # 1, 2, 4

1. (a) $\frac{dM}{dt} = 0.04M$
(b) $\frac{dM}{dt} = 1000 + 0.04M$

2. $\frac{dP}{dt} = kP(N - P)$,where k is the positive constant of proportionality.

4. $\frac{dG}{dt} = T - kG$,where k is the positive constant of proportionality.

Problem 1 from Handout F

1. (a) Take the sum of the number of calories in succesively larger spherical "shells." The equation for the surface area of a spherical shell is $4\pi r^2$,so:

$$\# \text{ of Calories} = 4\pi \int_0^R x^2 \rho(x) dx$$

(b) Take the sum of the number of calories in circular slices. Note that in this problem x is the distance from the top of the mold, which would normally be on our $y-$axis. So, if we take the height of each slice to be dx, then the radii are given by $y = \sqrt{R^2 - x^2}$, so the number of calories in each slice is given by $\pi(R^2 - x^2)\delta(x)dx$, and the integral is:

$$\# \text{ of Calories} = \pi \int_0^R (R^2 - x^2)\delta(x) dx$$