1. Show that $\int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}9\pi$ - in other words, show analytically that the area of a circle of radius 3 is 9π by doing the following:

We'd like to eliminate $\sqrt{9-x^2}$ by making a substitution that makes the integrand a perfect square. We will exploit the trig identity $\sin^2 t + \cos^2 t = 1$, or, equivalently, $9\sin^2 t + 9\cos^2 t = 9$ We know that $9 - 9\sin^2 t$ is a perfect square, so we'll use the substitution $x = 3\sin t$. Now we need to write the entire integral in terms of t.

a) If $x = 3\sin t$ then what is dx in terms of t? $dx = 3\cos(t)dt$

b) If $x = 3\sin t$ then what is $\sqrt{9-x^2}$ in terms of t? $\sqrt{9-x} = \sqrt{9-9\sin^2 t} = \sqrt{9\cos^2 t} = 3\cos t$

c) If $x = 3\sin t$ then what are the new endpoints of integration in terms of t? 0, $\frac{\pi}{2}$

d) Write the integral in terms of t. $\int_0^{\frac{\pi}{2}} 9\cos^2 t dt$

e) Evaluate the integral in (d). $\int_0^{\frac{\pi}{2}} 9\cos^2 t dt = 9(\frac{t}{2} + \frac{\sin(2t)}{4}|_0^{\frac{\pi}{2}}) = 9 * \frac{\pi}{4} = \frac{1}{4}9\pi$

f) Conclude that the area of a circle of radius 3 is 9π. Because t represents the angle, the area from $t = 0$ to $t = \frac{\pi}{2}$ is one quarter of a circle with radius 3. Thus, the full area of the circle is $4 * \frac{1}{4}9\pi = 9\pi$.