Assignment for Mathematics 1b Problem Set # 4

(This problem set is due on Wed. for MWF classes and Thursday for TTH classes)

Read §8.4.
Do: §8.3 #5, 9
§8.4 # 2, 12, 13, 19 (for 19, explain your reasoning clearly. There are several different lines of reasoning that can be used.)
Plus:

1. The Alternating Series Test says that if an infinite series is

   i) alternating
   ii) the magnitude of the terms is decreasing
   iii) the magnitude of the terms tends to zero

   then the series converges.

   If any one of these three conditions is not satisfied then we cannot conclude that the series converges. Below we will show that we can concoct a series that diverges if conditions (i) and (iii) are satisfied but (ii) is not. Your job is to show that each of conditions (i) and (iii) are necessary by providing an example of a series satisfying the other two conditions but diverging.

   Conditions (i) and (iii) are satisfied by the series

$$1-\frac{1}{2}+\frac{2}{2}-\frac{1}{3}+\frac{2}{3}-\frac{1}{4}+\frac{2}{4}-\cdots+\frac{2}{n}-\frac{1}{n}+\cdots$$

   but

$$1+\left(-\frac{1}{2}+\frac{2}{2}\right)+\left(-\frac{1}{3}+\frac{2}{3}\right)+\left(-\frac{1}{4}+\frac{2}{4}\right)+\cdots+\left(\frac{2}{n}-\frac{1}{n}\right)+\cdots$$

   can be written

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{n}+\cdots$$

   which is the harmonic series. The harmonic series diverges, so the series displayed diverges.