

Assignment for Mathematics 1b Problem Set # 3

(This problem set is due on Monday, 9/24/01, for MWF classes and Tuesday, 9/25/01 for TTh classes.)

Read §8.3 pp. 583-587.

Do: §8.3 # 1, 2, 3, 4, 10, 19, 20.

Plus:

Problem on p -series

In this problem you will learn about a family of series known as p -series. A p -series is a series of the form $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$.

1. If $p < 0$ then the series diverges by the n th term test. Explain.

2. Show that if $p > 1$ then $\int_1^\infty \frac{1}{x^p} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^p} dx$ is finite.

Show that if $0 < p < 1$ then $\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^p} dx = \infty$.

(You'll need to break this into two cases. Think about why.)

Conclude that $\int_1^\infty \frac{1}{x^p} dx$ diverges for $0 < p < 1$ and converges for $p > 1$.

3. Conclude from your work in parts (a) and (b) that $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ converges if $p > 1$ and diverges if $p < 1$.