Assignment for Mathematics 1b Problem Set # 1

(This problem set is due on Wed. for MWF classes and Thursday for TTH classes)

Read the first part of §8.2, pp. 573-576.

Do: §8.2 # 3,6 (where for these two problems the instructions "Graph both the sequence of terms and the sequence of partial sums on the same screen " is changed to "Plot the partial sums.") # 12, 16,33

Additional problems:

1. Determine whether or not the following sums are geometric. If the sum is geometric, express the sum in closed form.
   a) $\sum_{k=1}^{70}(\frac{1}{k})$ b) $\sum_{k=1}^{50}(\frac{1}{k})^2$ c) $\sum_{k=1}^{60}(\frac{1}{k})^k$ d) $\sum_{k=1}^{60}(1.01)^{k/12}$
2. Find the sum of each of the following.
   a) $\sum_{k=0}^{100}(\frac{1}{3})^k$ b) $\sum_{k=0}^{\infty}(\frac{1}{3})^k$ c) $\sum_{k=2}^{100}(\frac{1}{3})^k$ d) $\sum_{k=2}^{\infty}(\frac{1}{3})^k$

Question: Do the first few terms of the series affect whether or not the series converges? Do the first few terms of a convergent series affect its sum?

Some of our problem sets will include *think, and think again problems.* Here is the message about such problems for the student.

You will receive full credit on your homework for a thoughtful answer to such a question - regardless of whether you have answered it correctly. The following class you must resubmit an answer. The second time you will get credit only if your answer is correct. In other words, if you are correct the first time then you simply resubmit it for double credit. If you are wrong the first time, and correct the second time, you still get double credit. If you are wrong both times, you get credit only for having tried it the first time. Only those problems labeled "think, and think again" will be graded in this manner.

Think, and think again:

1. Suppose you know that the infinite series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ converges and that $a_k > 0$ for k any positive integer. Let $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. For each of the following statements, determine whether the statement must be true, could possibly be true, or must be false.

   1. $\lim_{n\to\infty} a_n = 0$
   2. $\lim_{n\to\infty} s_n = 0$
   3. There exists a number M such that $s_n < M$ for all n.
      (This is equivalent to saying that the partial sums are bounded.)
   4. $\sum_{k=5}^{\infty} a_k$ converges

2. Suppose you know that $\lim_{n\to\infty} b_n = 0$. Can you be sure that the infinite series
   $b_1 + b_2 + b_3 + \cdots + b_n + \cdots$ converges?