Solutions to the Integration Problem Preparation for the Technique Test

1. Perform the substitution $u = e^x$ followed by $u = \sin\theta$. The answer is $\frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}e^x\sqrt{1-e^{x^2}} + C$.
2. Perform integration by parts with $u = x, dv = \sec^2 x dx$. The answer is $x\tan x + \ln|\cos x| + C$.
3. Use the fact that $4x + 4 = 2(2x+1) + 2$. The answer is $2x + \ln|2x+1| + C$.
4. Perform the substitution $u = x^2$. The answer is $\frac{7}{2}e^{x^2} + C$.
5. Use the fact that $x^2 + 6x + 9 = (x+3)^2$. The answer is $-\frac{1}{x+3} + C$.
6. Since $\sin(x^3)$ is an odd function and the interval is symmetric around 0, the integral is zero. (Note that we would not want to find an antiderivative for this function.)
7. Perform the substitution $u = 3x + 4$. The answer is $\frac{2}{9}(31\sqrt{31} - 7\sqrt{7})$.
8. Integrate by parts twice and note that we obtain the original integrand. The answer is $\frac{1}{2}e^x(\sin x - \cos x) + C$.
9. Integrate by parts with $u = \ln x, dv = xdx$. The answer is $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$.
10. Use the trigonometric identity $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$. The answer is $\frac{1}{2}x - \frac{1}{28}\sin(14x) + C$.
11. Perform the substitution $u = x^4 + x^3 + x^2 + x + 1$. The answer is $\ln|x^4 + x^3 + x^2 + x + 1| + C$.
12. Use partial fractions. The answer is $\frac{9}{2}ln|x-1| - 15ln|x-2| - \frac{21}{2}ln|x-3| + C$
13. $\frac{1}{2}\sin(x^2) + C$ (substit.)
14. $x^2\sin x + 2x\cos x - 2\sin x + C$ (parts twice)
15. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(4n+1)*(2n)!}$
16. $\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$ Use trig identity $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$
17. $\frac{1}{2}x^2\sin(x^2) + \frac{1}{2}\cos(x^2) + C$
18. $\sin(x) - \frac{1}{3}\sin^3(x) + C$
19. 1
20. $x(\ln x)^2 - 2x\ln x + 2x + C$
21. $\frac{1}{3}(\ln x)^3 + C$
22. $\ln 3$
23. $\frac{1}{2}(x^2+1)\ln(x^2+1) - \frac{1}{2}(x^2+1) + C$ Your answer may differ by a constant, of course.
24. $x - 2\arctan x + C$
25. $x - \ln|x+1| + \ln|x-1| + C$ Note $1 + \frac{2}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$
26. $\frac{1}{2}\tan^{-1}(x^2) + C$
27. $\sqrt{x^2+1} + C$
28. $\frac{1}{9}\sqrt{9x^2-4} + C$
29. $\frac{1}{4}\ln|e^x - 2| - \frac{1}{4}\ln|e^x + 2| + C$ Substit followed by partial fractions
30. $\frac{1}{2}\ln|e^{2x} - 4| + C$ substit.
31. $2\sqrt{7} - 2\sqrt{5}$

32. $2ln|x-3| - ln|x+1| + C$
33. $x + 2\tan^{-1} x + C$
34. $2\arcsin(x/2) + \frac{1}{2}x\sqrt{4-x^2} + C$
35. $x\arctan(5x) - 10\ln(1+25x^2) + C$
36. $\frac{1}{3}e^{x^3} + C$
37. $\frac{5}{6}sin^2(3x) + C$
38. $-\frac{3}{2}\ln(\cos 2x)$
39. $e^{\tan x}$
40. $-\frac{\cos(2\sin x)}{2}$
41. $\frac{\ln(1+x)}{(1+x)} + \frac{1}{(1+x)}$
42. $\frac{x^2}{2}2x + 3ln|1+x| - 1/(1+x) + C$
43. $-\frac{2}{1+x}$
44. $\frac{\ln x}{2} - \frac{\ln(2x+4)}{2}$
45. $\frac{e^x \sin x - e^x \cos x}{2}$
46. $3x\sin x + 3\cos x$
47. $\frac{1}{3}(x^2+2x)^{\frac{3}{2}}$
48. $\int \cos(2x)e^{3x}dx = e^{3x}\left[\frac{2\sin(2x)+3\cos(2x)}{13}\right] + C$
49. $\int_1^e \frac{\sin(\ln(x))}{x}dx = 1 - \cos(1)$
50. $\int \sin(\ln(x))dx = \frac{x}{2}\big(\sin(\ln(x)) - \cos(\ln(x))\big) + C$
51. $\int_0^4 e^{\sqrt{x}}dx = 2e^2 + 2$
52. $\int \frac{3}{x^2(x^2+9)}dx = \frac{-1}{3x} - \frac{\arctan(x/3)}{27}$
53. $\int \frac{x^3}{x^2+1}dx = \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + C$
54. $\int \frac{dx}{x^3+x}dx = \ln(x) - \frac{1}{2}\ln(x^2+1) + C$
55. $\int \frac{\sin(x)dx}{\cos^2(x)-5\cos(x)+4}dx = \frac{1}{3}\ln\left|\frac{1-\cos(x)}{4-\cos(x)}\right| + C$
56. $\int \frac{dx}{e^x-1} = \ln|e^x - 1| - x + C$
57. $\int \frac{du}{(1+2u)(u^2+u)^{1/2}}du = \arctan(2\sqrt{u^2+u}) + C$
58. $\int_0^1 \frac{dx}{\sqrt{4-x^2}}dx = \arcsin(1/2)$
59. $2e^{\sqrt{x}}$
60. $\frac{cos^3(x)}{3} - cos(x)$
61. $\frac{1}{2}ln(x^2+1) + tan^{-1}(x)$
62. $e^{(x+1)^2}$
63. $\frac{sin(2x)}{2}$

64. $-\sqrt{1-x^2}$

65. $-\frac{1}{2}cos(x^2-1)$

66. $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}$

67. $\frac{(x^2+1)^5}{5}+\frac{2(x^2+1)^3}{3}+x^2+1$

68. 0

69. $2e^3/9+1/9$

70. $-\frac{1}{4}\cos(2x)+C$

71. $\ln\ln\ln x+C$

72. $\frac{2}{5}\sin x\,e^{2x}-\frac{1}{5}\cos x\,e^{2x}$

73. $-\frac{1}{2}\cos 2x-\frac{1}{6}\cos^3 2x+C$

74. $x\ln\sqrt{x}-x/2+C$ (begin with the substitution $x=u^2$.)

75. $\frac{2}{4}\ln|x-2|+\frac{1}{4}\ln|x+2|+C$

76. $\int\frac{1}{(x^2+1)^{3/2}}\,dx=\frac{x}{\sqrt{1+x^2}}$

77. $\int_1^2\frac{1}{x(1+x^3)}\,dx=\ln(x)-\frac{\ln(1+x^3)}{3}\Big|_1^2=\frac{2\ \ln(4/3)}{3}$

78. $\int\frac{\cos(x)}{1+\sin(x)}\,dx=\ln(1+\sin(x))$

79. $\int\frac{x^3+1}{x^2-1}\,dx=\frac{x^2}{2}-\ln(x-1)$

80. $\int\frac{1}{x(x^2+a^2)}\,dx=\frac{\ln(x)}{a^2}-\frac{\ln(x^2+a^2)}{2a^2}$

81. $\int_{1/2}^1\frac{1}{x^2\sqrt{1-x^2}}\,dx=\lim_{t\to 1}-\frac{\sqrt{1-x^2}}{x}\Big|_{1/2}^t=\sqrt{3}$

82. $\int_1^\infty\frac{\ln(x)}{x^2}\,dx=\lim_{t\to\infty}-\frac{1+\ln(x)}{x}\Big|_1^t=1$

83. $\int\frac{x}{(x^2-a^2)^{3/2}}\,dx=-\frac{1}{\sqrt{x^2-a^2}}$

84. $\frac{1}{2}\ln\left(\frac{2-\sqrt{2}}{3(\sqrt{2}+2)}\right)$

85. $\frac{x^2(\ln x)^2}{2}-\frac{x^2}{2}\ln x+\frac{x^2}{4}+C$

86. $3\left(\ln 2-\frac{1}{2}\right)$

87. $\frac{1}{2\sqrt{3}}\ln\left(\frac{\sqrt{3}-2}{3(\sqrt{3}+2)}\right)$

88. $\tan x-x+C$

89. $\frac{\pi}{12}$

90. $\frac{\pi}{2}$

91. $-\frac{1}{36}$

92. $\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{e}}\right)$

93. $\sqrt{x^2+3}+C$

94. $\int e^{lnx}dx=x^2/2+C$

95. $\int \frac{(x-1)dx}{(x^2+1)(x+1)} = 1/2\ln(x^2+1) - \ln|x+1| + C$

96. $\int_0^1 \sqrt{1-x^2}x^2 dx = \pi/16$

97. $\int \cos(x)\ln\sin(x)dx = t\ln t - t + C$, where $t = \sin x$.

98. $\int \frac{\sqrt{x}dx}{x+1} = 2(t - \arctan(t)) + C$, where $t = \sqrt{x}$

99. $\int \sqrt{\ln x}/x dx = (2/3)t^{3/2} + C$, where $t = \ln x$

100. $\int x^2 e^x dx = (x^2 - 2x + 2)e^x + C$

101. $\int (\sin x)^3 dx = (\cos x)^3/3 - \cos x + C$

102. $\int \ln(x^2+1)dx = x\ln(x^2+1) - 2x + 2\arctan(x) + C$

103. $\int \frac{\ln\ln x}{x} dx = t\ln t - t + C$, where $t = \ln x$