- Suppose $\sum_{k=1}^{\infty} a_k$ converges. Denote its sum by S. From Equation (H.1) we know

$$\int_1^n f(x)\,dx \le a_1 + a_2 + \cdots + a_{n-1}$$

$$\lim_{n\to\infty} \int_1^n f(x)\,dx \le \lim_{n\to\infty} \sum_{k=1}^{n-1} a_k = S.$$

  If $\lim_{n\to\infty} \int_1^n f(x)\,dx$ is bounded, so too is $\lim_{b\to\infty} \int_1^b f(x)\,dx$ (given the hypotheses).

- Suppose $\sum_{k=1}^{\infty} a_k$ diverges. Because the terms are all positive, we know $\lim_{n\to\infty} \sum_{k=1}^{n} a_k = \infty$. From Equation (H.1) we know

$$a_2 + \cdots + a_n \le \int_1^n f(x)\,dx$$

$$\lim_{n\to\infty} \sum_{k=2}^{n} a_k \le \lim_{n\to\infty} \int_1^n f(x)\,dx.$$

  We conclude that $\lim_{b\to\infty} \int_1^b f(x)\,dx = \infty$; the improper integral diverges.