

Handout J

1. Solve the following differential equations for $y(x)$.

(a) $y'' - 9y' = 0$

(b) $y'' - 9y = 0$

(c) $y'' + 9y = 0$

(d) $y'' - 9 = 0$

(e) $y'' - 2y' - y = 0$

(f) $y'' - 2y' + 2y = 0$

2. Suppose that $x'' + bx' + cx = 0$ is used to model the position of a block at the end of a vibrating spring.

(a) What can you say about the signs of b and c ? Explain.

(b) As long as friction plays a role, we expect that regardless of the initial conditions $\lim_{t \rightarrow \infty} x(t) = 0$. Explain how your answer to part (a) guarantees this.

Hint: it is necessary to do three different cases.

3. Write a differential equation of the form $x'' + bx' + cx = 0$ such that if $x(0) = 1$ and $x'(0) = 2$ then $x(t)$ has the property that

(a) $\lim_{t \rightarrow \infty} x(t) = 0$

(b) $\lim_{t \rightarrow \infty} x(t) = \infty$

(c) $\lim_{t \rightarrow \infty} x(t)$ does not exist.

Note: there are not unique answers to these problems!