Handout H

1. Show that if y_1 and y_2 are both solutions to the differential equation $y'' + by' + cy = 0$ then $y = C_1y_1 + C_2y_2$ is also a solutions to $y'' + by' + cy = 0$.

Addition to Wednesday's or Thursday's assignment (due Friday or Tuesday):

1. Consider the following model for the population levels of predators and prey, in a given enviroment at time t.

$$\frac{dx}{dt} = ax - bx^2 - cxy$$

$$\frac{dy}{dt} = -dy + exy$$

All the parameters a, b, c, d, e are positive constants.

(a) Which of x and y are the predators (and prey)? Explain please.

(b) If you started of with some prey and no predators, what do you expect in the long run?
Suppose, on the other hand, you start with some predators and no prey. What happens in the long run?

(c) Show that there is an equilibrium for the system at the point $(x, y) = \left(\frac{d}{e}, \frac{a}{c} - \frac{bd}{ce}\right)$.

(d) Give some interpretation to the parameters a, b, c, d, and e.

2. Consider the following model for the populations of two species in competition in a given enviroment at time t (say,.. mice and rats).

$$\frac{dx}{dt} = x - x^2 - axy$$

$$\frac{dy}{dt} = y - y^2 - axy$$

The parameter a is a positive constant.

(a) If you started out with some mice, no rats, what would you expect to see in the long run? (What about some rats, no mice)

(b) Sketch the phase portrait (label the equilibrium points and null clines) of the system for the cases $a = 1/2$ and $a = 3/2$ and note the difference in the behavior of the system. Please describe the difference between the cases $a < 1$ and $a > 1$.