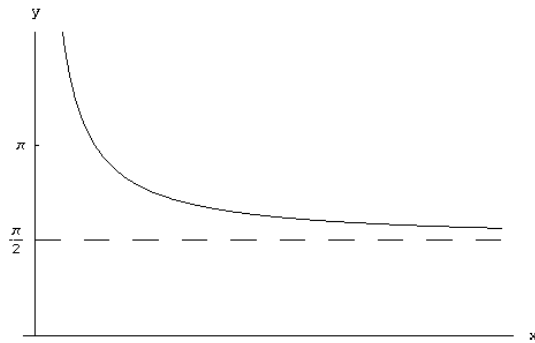


1. *Extra Credit:* An object is dropped from a building and encounters air resistance proportional to the square of its velocity.
 - a) Write a differential equation describing the velocity $v(t)$ of the object at time t . (Hint: p. 987 of the differential equations handout contains the simpler case of no air resistance.)
 - b) Solve this differential equation for $v(t)$. What is the initial condition? Find the solution that satisfies this initial condition. (Hint: You can use partial fractions and you'll have lots of logs in your answer.)
 - c) Solve for v .
 - d) (Extra Extra Credit) Compute the total distance $d(t)$ that the object has fallen at time t .

2. Why can't the graph below be a solution to $y' + \cos y = y$?



3. A canister contains 10 liters of blue paint. Paint is being used at a rate of 2 liters per hour and the canister is being replenished by pale blue paint that is 80% blue paint and 20% white paint. Assume the canister is well-mixed and that paint is both entering and leaving the canister continuously
 - a) Write a differential equation whose solution is $w(t)$, the amount of white paint in the canister at time t . Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.
 - b) Write a differential equation whose solution is $b(t)$, the amount of blue paint in the canister at time t . Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.

4. Let $x = x(t)$ be the number of thousands of beasts of species X at time t .
Let $y = y(t)$ be the number of thousands of beasts of species Y at time t .

Suppose that

$$\frac{dx}{dt} = .1x - .05xy$$

$$\frac{dy}{dt} = .1y - .05xy$$

- (a) Is the interaction between species X and Y symbiotic, competitive, or a predator/prey relationship?
- (b) What are the equilibrium populations?
- (c) Find the nullclines and draw directed horizontal and vertical tangents in the phase plane. (Consult the supplement for illustrations.)
- (d) The nullclines divide the first quadrant of the phase plane into four regions. In each region, determine the general directions of the trajectories.
- (e) If $x = 0$, what happens to $y(t)$? How is this indicated in the phase plane?
If $y = 0$, what happens to $x(t)$? How is this indicated in the phase plane?
- (f) Use the information gathered in parts (b) - (e) to sketch representative trajectories in the phase plane. Include arrows indicating the direction in which the trajectories are traveled.
- (g) For each of the initial conditions given below, describe how the size of the populations of X and Y change with time and what the situations will look like in the long run.
 - i. $x(0) = 2 \quad y(0) = 1.8$
 - ii. $x(0) = 2 \quad y(0) = 2.3$
 - iii. $x(0) = 2.2 \quad y(0) = 2$
- (h) Does this particular model support or challenge Charles Darwin's Principle of Competitive Exclusion?