Additional Problems for Mathematics 1b
Handout A

1.) The Alternating Series Test says that if an infinite series is

i) alternating

ii) the magnitude of the terms is decreasing

iii) the magnitude of the terms tends to zero

then the series converges.

If any one of these three conditions is not satisfied then we cannot conclude that the series converges. Below we will show that we can concoct a series that diverges if conditions (i) and (iii) are satisfied but (ii) is not. Your job is to show that each of conditions (i) and (iii) are necessary by providing an example of a series satisfying the other two conditions but diverging.

Conditions (i) and (iii) are satisfied by the series

$$1 - \frac{1}{2} + \frac{2}{2} - \frac{1}{3} + \frac{2}{3} - \frac{1}{4} + \frac{2}{4} - \cdots + \frac{2}{n} - \frac{1}{n} + \cdots$$

but

$$1 + \left(-\frac{1}{2} + \frac{2}{2}\right) + \left(-\frac{1}{3} + \frac{2}{3}\right) + \left(-\frac{1}{4} + \frac{2}{4}\right) + \cdots + \left(\frac{2}{n} - \frac{1}{n}\right) + \cdots$$

can be written

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{n} + \cdots$$

which is the harmonic series. The harmonic series diverges, so the series displayed diverges.

2.) What EXACTLY do we mean when we write $\sum_{n=0}^{\infty} a_n = 7$? Your answer can be brief, but must be precise and accurate. You will get full credit only if you use words correctly.

3.) Suppose that a power series of the form $\sum_{n=0}^{\infty} c_n(x-1)^n$ has a radius of convergence of 5. What are the possibilities for the interval of convergence of the series?

4.) Suppose that the interval of convergence of the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ is $(-3, 1]$.
(a) What is a?
(b) Does the series converge for $x = -2.8$? For $x = 2.8$?