1. i) $x>0$, so divide by x: $\frac{dy}{dx} + \frac{2}{x}y = x^2$

Find integrating factor: $\mu(x) = e^{\int \frac{2}{x}dx} = e^{2\ln(x)} = e^{\ln(x^2)} = x^2$.

So $x^2 \frac{dy}{dx} + 2xy = x^4 \Rightarrow [x^2 \cdot y]' = x^4 \Rightarrow x^2 \cdot y = \frac{x^5}{5} + C$

$$\boxed{y = \frac{x^3}{5} + \frac{C}{x^2}}$$

ii) $y(1) = 1 = \frac{1^3}{5} + \frac{C}{1^2} = \frac{1}{5} + C \Rightarrow C = 4/5$

So particular sol'n is $\boxed{y(x) = \frac{x^3}{5} + \frac{4}{5x^2}}$

2. i) Use Ratio Test on $\sum \frac{n^\pi}{\pi^n}$

$L = \lim_{n\to\infty} \left|\frac{(n+1)^\pi}{\pi^{n+1}}\right| \cdot \left|\frac{\pi^n}{n^\pi}\right| = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^\pi \cdot \frac{1}{\pi} = \frac{1}{\pi} < 1$ So $\boxed{\text{Abs. Conv.}}$

ii) $\frac{\ln(n)}{n} > \frac{1}{n}$ for $n \geq 3$. Since $\sum \frac{1}{n}$ is a p-series with $p=1$, it diverges.

So, by Comp. Test, $\sum \frac{\ln(n)}{n}$ diverges.

So answer is not abs. conv. Check if cond. conv.

a) $\lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$

l'Hopital's

b) let $f(x) = \frac{\ln(x)}{x}$; $f'(x) = \frac{x\cdot\frac{1}{x} - \ln(x)\cdot 1}{x^2}$ $f'(x) < 0$ when $\ln(x) \geq 1$ or $x \geq 3$.

So $f(x)$ is eventually decreasing.

By Alt. Series test, $\boxed{\text{Cond. Conv.}}$

iii) Use limit comparison test with $\sum \frac{1}{n^{5/2}}$, this is p-series with $p = 5/2 > 1$

So $\boxed{\text{Abs. Conv}}$

iv) $\lim_{n\to\infty} \frac{n^{26} - 53n^{19} + 11n^7 - 21}{66 - 92n^{26}} = \frac{-1}{92} \neq 0$ so $\boxed{\text{diverges}}$ by div. test

v) $\frac{1}{5} - \frac{1}{n^2}$ is positive (for $n \geq 3$) & less than 1.

So $\left(\frac{1}{5} - \frac{1}{n^2}\right)^{n^3} < \left(\frac{1}{5} - \frac{1}{n^2}\right)^n < \frac{1}{5^n}$. $\sum \frac{1}{5^n}$ is geometric series with $r = \frac{1}{5} < 1$

So conv. By Comp. test $\boxed{\text{Abs Conv.}}$