§7.1 Integration by Parts

The product rule, $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$, gives us:

$$\int u\,dv = uv - \int v\,du$$

When integrating by parts, watch out for a bunch of things.

1. Choose what you want u and dv to be carefully
   Some students recommend using the acronym LIPET (logs, inverse trig, polynomials, exponentials, trig functions)
   Let the part of the integrand that appears first in LIPET be u.
2. If you have to integrate by parts several times, let the same sort of function be u each time (like let $u = e^x$ each time)
3. Be careful with the minus sign $uv - \int v\,du$ and remember to distribute if you integrate several times
4. Remember the chain rule.
   The antiderivative of $\frac{1}{\sin x}$ is NOT $\ln|\sin x|$ since
   $\frac{d}{dx}\ln|\sin x| = \frac{1}{\sin x}\cdot\frac{d(\sin x)}{dx} = \frac{\cos x}{\sin x}$

Look at problems 9, 23, 31, 47

§7.2 Trigonometric Integrals

The point of these is to make nonobvious trig integrals look easier.
Some identities you may want to know (or at least be familiar with):

$$\sin A\cos B = \tfrac{1}{2}[\sin(A-B) + \sin(A+B)]$$

$$\sin A\sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A\cos B = \tfrac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin(x+y) = \sin x\cos y + \sin y\cos x$$
$$\longrightarrow \sin(2x) = 2\sin x\cos x$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\longrightarrow \cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin^2 x + \cos^2 x = 1$$
$$\longrightarrow 1 + \cot^2 x = \csc^2 x$$
$$\longrightarrow \tan^2 x + 1 = \sec^2 x$$