§9.3 Separable Equations

<u>Definition</u>: A <u>separable equation</u> is a first order Diff EQ that can be factored into $\frac{dy}{dx} = g(x)f(y) = \frac{g(x)}{h(y)}$ (if $f(y) \neq 0$)

$\longrightarrow h(y)\,dy = g(x)\,dx$

$\longrightarrow \int h(y)\,dy = \int g(x)\,dx$

Now, if we can integrate both sides, we can solve for y in terms of x.

<u>Definition</u>: When you integrate and get $y = (\text{function of } x) + C$, the <u>family of curves</u> is the set of all functions you get for y as C ranges over the real numbers.

<u>Definition</u>: An <u>orthogonal trajectory</u> of a family of curves is a curve that intersects every curve in the family orthogonally (perpendicularly).

So if you have an equation $y' = (\text{stuff})$ for the family, solve $y' = \frac{-1}{(\text{stuff})}$ to find the orthogonal trajectories.

<u>Mixing Problems</u> to solve for the amount of substance (salt, for example) in the solution at time t, $y(t)$, remember that $\frac{dy}{dt} = (\text{rate the salt goes in}) - (\text{rate the salt goes out})$

Look at problems 11, 25, 31, 39

§9.4 Exponential Growth and Decay

<u>Example</u>: Radioactive decay $\longrightarrow m'(t) = k\,m(t)$ where $k < 0$

We can also compute half-life from this

The initial value problem $\frac{dy}{dt} = ky$, $y(0) = y_0$, has solution $y(t) = y_0 e^{kt}$

<u>Interest</u>: A rate of r, compounded n times a year, initial amount A_0:

$A(t) = A_0\,(1 + \frac{r}{n})^{nt}$

Continuously compounded: $A(t) = A_0 e^{rt}$

Look at problems 3, 7, 9, 17