SOLNS: 9.7 PREDATOR–PREY SYSTEMS

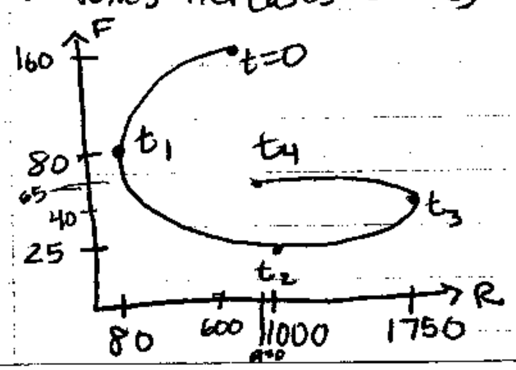
① ⓐ x = predator (If $y=0$, $dx/dt = -.05x$, so in absence of prey, x declines @ rate $\propto x$.)
Growth of prey population, $.1y$, is restricted only by encounters with predators (the $-.005xy$ term).
The predator population increases only through $.0001xy$ term, ie, food only from prey, not additional food sources

ⓑ y = predator (If $x=0$, $dy/dt = -.015y$, so in absence of prey, y declines @ rate $\propto y$)
Growth of prey population, $.2x$, is restricted not only by encounters with predators ($-.006xy$ term) but also by a carrying capacity, represented by the implicit $(1 - x/1000)$ factor in $.2x - .0002x^2$.
The predator population increases only through the prey encounters ($.00008xy$ term), not additional food sources.

② ⓐ Cooperation model. (Consider the xy terms) Note that an increase in y makes dx/dt larger ($+.0001xy$ term), and an increase in x makes dy/dt larger ($+.00004xy$ term).

ⓑ Competition model. (Consider the xy terms) Note that an increase in x reduces growth rate of y, by $-.0002xy$ term; an increase in y reduces growth rate of x, by $-.0006xy$ term. (Also note that x, y have carrying capacities of 750, 2500, respectively:
$.15x - .0002x^2 = .15x(1 - x/750)$,
$.2y - .00008y^2 = .2y(1 - y/2500)$.

④ At $t=0$, there are ≈ 600 rabbits, 160 foxes. At $t=t_1$, rabbits reach minimum population of 80; # of foxes is also ≈ 80. At $t=t_2$, # foxes is minimum of 25, while # rabbits rebounds to 1000. At $t=t_3$, # foxes increases to 40, rabbit population reaches maximum of 1750. At $t=t_4$, # foxes increases to 65, # rabbits decreases to 950.



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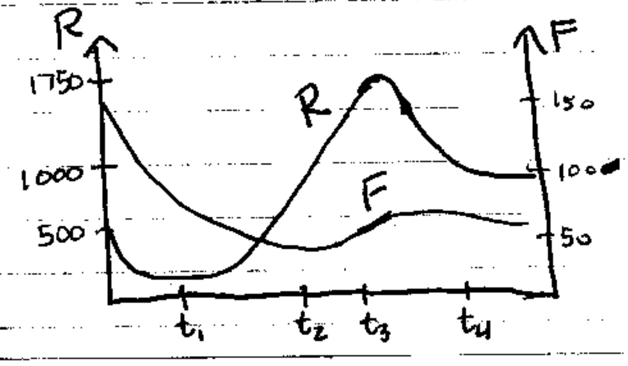
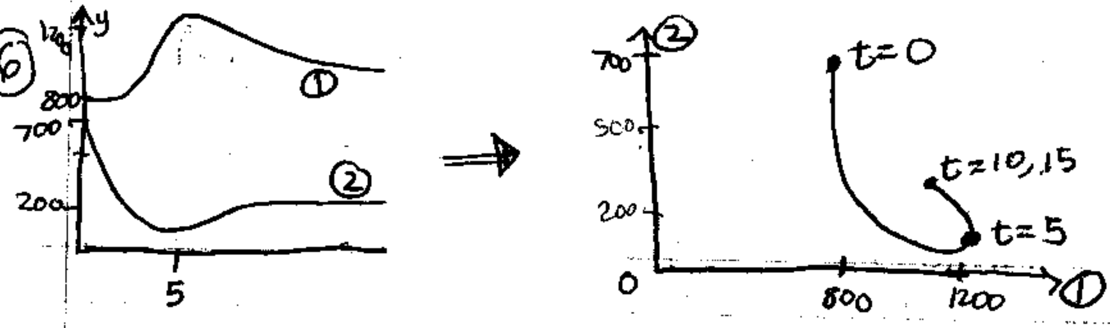


(6)



(8) (a) Equilibrium soln's means $\frac{dA}{dt} = \frac{dL}{dt} = 0$

So $2A - .01AL = 0 \rightsquigarrow A = 0$ or $2 - .01L = 0 \rightsquigarrow L = 200$

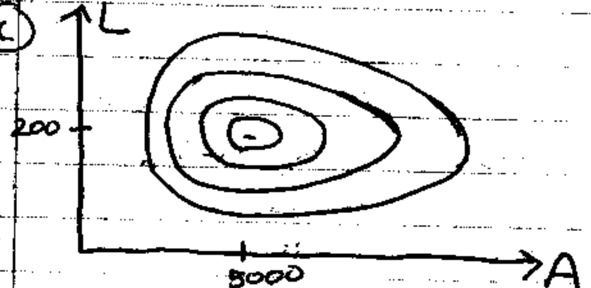
$-.5L + .0001AL = 0 \rightsquigarrow L = 0$ or $-.5 + .0001A = 0 \rightsquigarrow A = 5000$

trivial soln: $(A, L) = (0, 0)$. If there are no aphids/ladybugs, there population will not change

nontrivial soln: $(A, L) = (5000, 200)$, so for this population, the populations still will not change

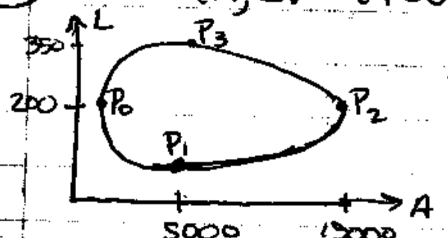
(b) $$\frac{dL}{dA} = \frac{dL/dt}{dA/dt} = \frac{-.5L + .0001AL}{2A - .01AL}$$

(c)



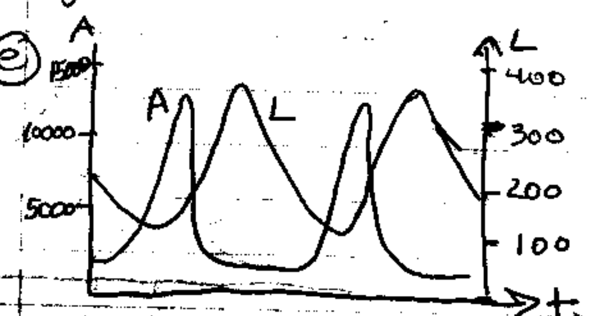
Note that the phase trajectories (solution curves) are all closed curves, that have the equilibrium point (5000, 200) inside them.

(d) $t = 0$: $(A, L) = (1000, 200)$



At P_0 (1000, 200) $dA/dt = 0$, $dL/dt = -80$, so # of ladybugs is decreasing (so we proceed counterclockwise). At P_0, there aren't enough aphids to support the ladybug population, so the # of ladybugs decreases, & # aphids begins increasing. At P_1 (5000, 100) ladybug population reaches minimum, while aphid population rapidly increases. At P_2 (14250, 200), aphid population reaches maximum, but for $P_2 \to P_3$, increasing ladybug population depletes aphid population. At P_3, ladybugs reach maximum, depleting aphid population, but on $P_3 \to P_0$, decreasing aphid numbers support fewer ladybugs, decreasing ladybug population. Cycle continues from P_0 as before.

(e)



Both graphs have same period, and graph of L peaks approximately 1/4 of a cycle after the graph of A peaks.