Section 9.3

(6) $y' = \dfrac{xy}{2\ln y}$

$$\int \frac{2\ln y}{y}\,dy = \int x\,dx$$

$$(\ln y)^2 = \frac{x^2}{2} + C$$

$$\ln y = \pm\sqrt{x^2/2 + C}$$

$$\boxed{y = e^{\pm\sqrt{\frac{x^2}{2} + C}}}$$

(12) $x + 2y\sqrt{x^2+1}\,\dfrac{dy}{dx} = 0$ $\qquad y(0) = 1$

$$x\,dx + 2y\sqrt{x^2+1}\,dy = 0$$

$$\int 2y\,dy = -\int \frac{x\,dx}{\sqrt{x^2+1}}$$

$$y^2 = -\sqrt{x^2+1} + C$$

$1 = -1 + C \qquad \boxed{C = 2}$

thus, $\boxed{y^2 = 2 - \sqrt{x^2+1}}$

Section 9.3

(16) $\frac{dy}{dx} = \frac{y^2}{x^3}$, $y(1) = 1$

$$\int \frac{dy}{y^2} = \int \frac{dx}{x^3}$$

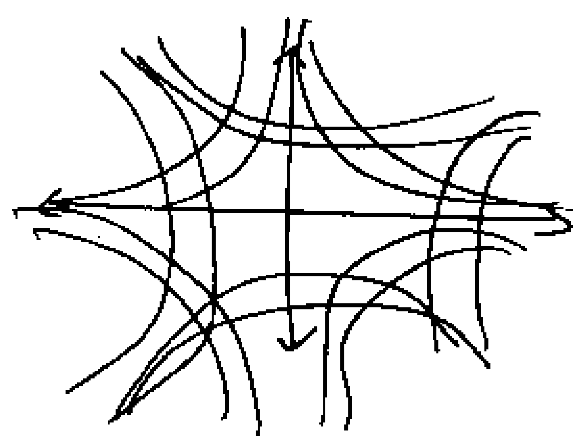
$$-\frac{1}{y} = -\frac{1}{2x^2} + C$$

$$-1 = -\frac{1}{2} + C \quad \boxed{C = -\frac{1}{2}}$$

Thus, $\frac{1}{y} = \frac{1}{2x^2} + \frac{1}{2}$, $\boxed{y = \frac{2x^2}{x^2+1}}$

(24) The curves $x^2 - y^2 = k$ are hyperbolas.
The orthogonal trajectories must satisfy
$y' = -y/x$, $\frac{dy}{y} = -\frac{dx}{x}$, $\ln|y| = -\ln|x| + C$,

$\underline{xy = C}$



Section 9.3

(28) from 9.2.28: $\frac{dy}{dt} = -\frac{1}{50}(y-20)$

$$\int \frac{dy}{y-20} = \int \left(-\frac{1}{50}\right) dt$$

$$\ln(y-20) = -\frac{1}{50}t + C$$

$$y - 20 = Ke^{-t/50}$$

$$y(t) = Ke^{-t/50} + 20$$

$y(0) = 95$

$95 = K + 20$

$\boxed{K = 75}$

$\boxed{y(t) = 75e^{-t/50} + 20}$

(32) a) The amount of new currency introduced per day is $\frac{dx}{dt} = \frac{10-x}{10} \cdot 0.05$

$\boxed{= 0.005(10-x)}$ billion \$ per day

b) $\frac{dx}{10-x} = 0.005\,dt$

$$\int \frac{-dx}{10-x} = \int -0.005\,dt$$

$$\ln(10-x) = -0.005t + C$$

$$10 - x = Ce^{-0.005t}$$

$C = e^c$

from $x(0) = 0$, $C = 10$

$\boxed{x(t) = 10(1 - e^{-0.005t})}$

c) \$9 billion $= 10(1 - e^{-0.005t})$

$t = 200(\ln 10)$

$\boxed{= 460.5 \text{ days} = 1.26 \text{ years}}$

Section 9.3

(34) a)

$$\frac{dy}{dt} = \left(0.05\ \frac{kg}{L}\right)\left(5\frac{L}{min}\right) + \left(.04\frac{kg}{L}\right)\left(10\frac{L}{min}\right) - \left(\frac{y(t)}{1000}\ \frac{kg}{L}\right)\left(15\frac{L}{min}\right)$$

$$\boxed{= \frac{130-3y}{200}\ \frac{kg}{min}}$$

now

$$\int \frac{dy}{130-3y} = \int \frac{dt}{200}$$

$\downarrow$ solve ~~[illegible]~~

$$130 - 3y = 130\,e^{-3t/200}$$

$$\boxed{y = \frac{130}{3}\left(1 - e^{-3t/200}\right) kg}$$

b) after an hour:

$$y = \frac{130}{3}\left(1 - e^{-180/200}\right) \boxed{= 25.7\ kg}$$