Section 9.1

② $y = \dfrac{2 + \ln x}{x}$

$y' = \dfrac{x\left(\frac{1}{x}\right) - (2+\ln x)(1)}{x^2} = \dfrac{-1 - \ln x}{x^2}$ & $y(1) = \dfrac{2+\ln 1}{1} =$

SUB IN! SUB IN!

$x^2 y' + xy = x^2\left(\dfrac{-1-\ln x}{x^2}\right) + x\left(\dfrac{2+\ln x}{x}\right)$

$= (-1 - \ln x) + (2 + \ln x) = 1$ ✓

④ $y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$y'' + y' - 6y = 0$

$r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$

$(r^2 + r - 6)e^{rt} = 0$

FACTOR! $(r+3)(r-2) = 0$

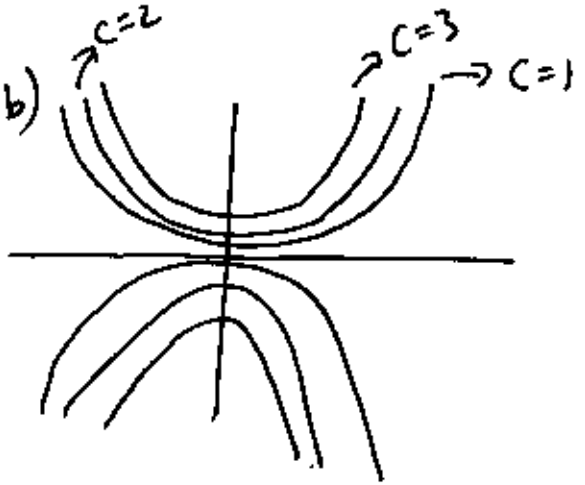
$\boxed{r = -3, 2}$

Section 9.1

⑥ a) $y = Ce^{x^2/2}$

$y' = Ce^{x^2/2}\left(\frac{2x}{2}\right) = xCe^{x^2/2} = xy$

b)



c) $y(0) = 5$, $Ce^0 = 5$, $C = 5$

thus, solution → $\boxed{y = 5e^{x^2/2}}$

d) $y(1) = 2$, $Ce^{1/2} = 2$, $C = 2e^{-1/2}$

thus, solution → $y = 2e^{-1/2}e^{x^2/2}$

$= 2e^{(x^2-1)/2}$

Section 9.1

(8) a) if x is close to zero, slope must be nearly horizontal.

if x is large, slope must be nearly vertical.

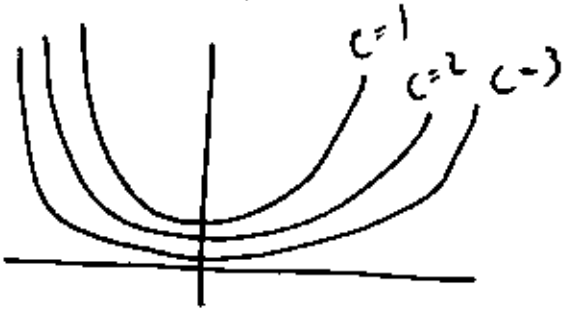
b) $y = (C - x^2)^{-1/2}$

$y' = x(C - x^2)^{-3/2}$

$xy^3 = x\left[(C-x^2)^{-1/2}\right]^3$

$= x(C - x^2)^{-3/2} = y'$

c)



d) $y(0) = (C-0)^{-1/2} = 1/\sqrt{C}$, $y(0) = 2$

$\sqrt{C} = \frac{1}{2}$, $C = \frac{1}{4}$

thus, $\boxed{y = \left(\frac{1}{4} - x^2\right)^{-1/2}}$

(10) a) $y = k$, $y' = 0$, thus, $\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$

$0 = k^4 - 6k^3 + 5k^2$

$k^2(k^2 - 6k + 5) = 0$

$k^2(k-1)(k-5) = 0$

$k = 0, 1, \text{ or } 5$

b) y is increasing

c) y is decreasing

Section 9.1

(12) a) the coffee cools most quickly right when it's removed from the heat source.

rate → 0

coffee → room temp

b) $\frac{dy}{dt} = k(y - R)$

k → constant

y → temp. of coffee

R → room temp.

the model is appropriate!

c)