Solution Set – Section 7.8

Math 16
Fall 2000

#6, 24, 36, 52, 58, 68

6) $$\int_{-\infty}^{0} \frac{1}{2x-5}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty} \frac{1}{2}\ln|2x-5|\Big|_t^0$$

$$= \frac{1}{2}\ln 5 - \underline{\underline{\infty}}$$, so this integral $\boxed{\text{diverges}}$.

Here we're taking the ln of an infinite quantity!

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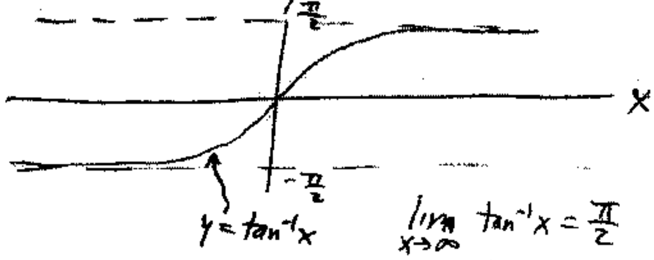
24) $$\int_{-\infty}^{\infty} \frac{1}{r^2+4}\,dr = \lim_{t\to\infty}\int_{-t}^{t} \frac{1}{r^2+4}\,dr = \lim_{t\to\infty}\int_{-t}^{t} \frac{2}{4(r^2+1)}\,du$$

$r = 2u$
$dr = 2\,du$

$$= \lim_{t\to\infty} \frac{1}{2}\tan^{-1}(2u)\Big|_{-t}^{t}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \boxed{\frac{\pi}{2}}$$



$$\lim_{x\to\infty}\tan^{-1}x = \frac{\pi}{2}$$
$$\lim_{x\to-\infty}\tan^{-1}x = \frac{-\pi}{2}$$

(so this converges, since it has a finite limit)

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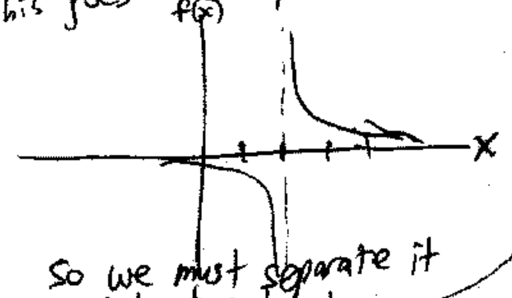
36) $$\int_0^4 \frac{1}{x^2+x-6}\,dx = \int_0^4 \frac{1}{(x-2)(x+3)}\,dx$$

This goes to infinity at x=2:



so we must separate it into two limits:

$$= \lim_{t\to2^-}\int_0^t \frac{1}{(x-2)(x+3)}\,dx + \lim_{t\to2^+}\int_t^4 \frac{1}{(x+2)(x+3)}\,dx$$

Let's take this first limit:

$$\frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

$$1 = A(x+3) + B(x-2)$$

$$\left.\begin{array}{l} A+B=0 \\ 3A-2B=1 \end{array}\right\} \longrightarrow \begin{array}{l} A=\frac{1}{5} \\ B=\frac{-1}{5} \end{array}$$

$$\frac{1}{5}\lim_{t\to2^-}\int_0^t \frac{1}{x-2} - \frac{1}{x+3}\,dx = \frac{1}{5}\lim_{t\to2^-}\left(\ln|x-2| - \ln|x+3|\,\Big|_0^t\right)$$

$$= \frac{1}{5}\left(\text{"}\ln\infty\text{"} - \cancel{\ln t} - \ln 2 + \ln 5\right).$$ This is infinite! Therefore, this integral (just from 0 to 2) diverges; therefore the entire integral must diverge.

52) $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$?

We know $\int_1^\infty \frac{1}{x}dx$ diverges. In this range, $\sqrt{x} \leq x$, so $\frac{1}{\sqrt{x}} \geq \frac{1}{x}$,
So we know $\int_1^\infty \frac{1}{\sqrt{x}}dx$ diverges by the Comparison test.
In this range, $\sqrt{1+\sqrt{x}} > 1$, so $\frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} > \frac{1}{\sqrt{x}}$,
so $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$ $\boxed{\text{diverges}}$.

58) $\int_e^\infty \frac{1}{x(\ln x)^p}\,dx = \int_1^\infty \frac{1}{u^p}\,dp = \int_1^\infty u^{-p}\,dp = \lim_{t\to\infty}\int_1^t u^{-p}\,dp$

$u = \ln x \quad du = \frac{1}{x}dx$

$= \lim_{t\to\infty} \frac{1}{-p+1}u^{-p+1}\Big|_1^t = \lim_{t\to\infty}\left(\frac{1}{-p+1}t^{-p+1} - \frac{1}{-p+1}\cdot 1\right)$

This converges only when $\lim_{t\to\infty} t^{-p+1}$ is finite, ie. $-p+1<0$

$\boxed{p>1}$

When $p>1$, $\lim_{t\to\infty} t^{-p+1} = 0$, so the limit is:

$\cancel{\frac{1}{-p+1}(0)} - \frac{1}{-p+1}\cdot 1 = \boxed{\frac{1}{p-1}}$

68) mean life:

$M = -k\int_0^\infty t e^{kt}\,dt = \cancel{-k}\left(\frac{t}{\cancel{k}}e^{kt} - \int_0^\infty \frac{1}{\cancel{k}}e^{kt}\,dt\right) = \lim_{n\to\infty}\left(-te^{kt} + \frac{1}{k}e^{kt}\right)\Big|_0^n$

$u = t \quad dv = e^{kt}dt$
$du = dt \quad v = \frac{1}{k}e^{kt}$

Since k is negative, $\lim_{n\to\infty} e^{kn} = 0$

so $M = \lim_{n\to\infty}\left(\cancel{-ne^{kn}} + \cancel{\frac{1}{k}e^{kn}} + \cancel{0e^{k\cdot 0}} - \frac{1}{k}e^{k\cdot 0}\right) = \boxed{\frac{-1}{k}}$

For ^{14}C, $k = -.000121$, so $M = \frac{-1}{k} = \frac{-1}{-.000121} = \boxed{8264 \text{ years}}$