Solution Set: Section 7.7

Math 16
Fall 2000

6) $\int_0^1 e^{-\sqrt{x}}\,dx$. $n=6 \to \Delta x = \frac{1}{6}$

$f(x) = e^{-\sqrt{x}}$

a) Midpoint Rule:

$$\int_0^1 e^{-\sqrt{x}}\,dx \approx \frac{1}{6}\left[f\left(\tfrac{1}{12}\right)+f\left(\tfrac{3}{12}\right)+f\left(\tfrac{5}{12}\right)+f\left(\tfrac{7}{12}\right)+f\left(\tfrac{9}{12}\right)+f\left(\tfrac{11}{12}\right)\right]$$

$$= .551740$$

b) Simpson's Rule:

$$\approx \frac{1}{6}\cdot\frac{1}{3}\left[f(0)+4f\left(\tfrac{1}{6}\right)+2f\left(\tfrac{2}{6}\right)+4f\left(\tfrac{3}{6}\right)+2f\left(\tfrac{4}{6}\right)+4f\left(\tfrac{5}{6}\right)+f(1)\right]$$

$$= .533979$$

The real answer:

$$\int_0^1 e^{-\sqrt{x}}\,dx = \int_0^{-1} 2ue^u\,du = 2ue^u - \int_0^{-1} 2e^u\,du$$

$u = -\sqrt{x}$, $x = u^2$, $dx = 2u\,dx$

$f = 2u$, $dg = e^u du$, $df = 2du$, $g = e^u$

$$= 2ue^u - 2e^u\Big|_0^{-1}$$

$$= 2e^u(u-1)\Big|_0^{-1}$$

$$= -4e^{-1}+2 \approx .528482$$

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8) $\int_0^{1/2} \sin(x^2)\,dx$ $n=4 \to \Delta x = \frac{1}{8}$

a) Trapezoidal Rule:

$$\approx \frac{1}{8}\cdot\frac{1}{2}\left[\sin(0)+2\sin\left(\left(\tfrac{1}{8}\right)^2\right)+2\sin\left(\left(\tfrac{1}{4}\right)^2\right)+2\sin\left(\left(\tfrac{3}{8}\right)^2\right)+\sin\left(\left(\tfrac{1}{2}\right)^2\right)\right]$$

$$= .042743$$

b) Simpson's Rule:

$$\frac{1}{8}\cdot\frac{1}{3}\left[\sin(0)+4\sin\left(\left(\tfrac{1}{8}\right)^2\right)+2\sin\left(\left(\tfrac{1}{4}\right)^2\right)+4\sin\left(\left(\tfrac{3}{8}\right)^2\right)+\sin\left(\left(\tfrac{1}{2}\right)^2\right)\right]$$

$$= .041478$$

24) $\int_0^1 e^{x^2}dx$

Error formula: $E \le \frac{K(b-a)^5}{180n^4}$ when $|f^{(4)}(x)| \le K$

Let's find K:

$f(x) = e^{x^2}$
$f'(x) = 2xe^{x^2}$
$f''(x) = e^{x^2}(2+(2x)2x) = e^{x^2}(2+4x^2)$
$f'''(x) = e^{x^2}(8x+(2+4x^2)2x) = e^{x^2}(12x+8x^3)$
$f^{(4)}(x) = e^{x^2}(12+16x^2+(12x+8x^3)2x) = e^{x^2}(12+40x^2+16x^4)$

K = maximum value of $|f^{(4)}(x)|$, which is $f^{(4)}(1)$
(since f is increasing over this entire range).
$= e(12+40+16) = 68e$

$b-a=1$

$E \le \frac{K(b-a)^5}{180n^4} \longrightarrow \frac{68e\,(1)}{180n^4} = .00001$

$n^4 = 102690$
$n = 17.9$

But n has to be an even integer, so the minimum n is $\boxed{18}$.

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34) $x = \int \frac{dx}{dt}\,dt = \int v\,dt$

$\Delta x = .5$

$x \approx (.5)(\frac{1}{3})[v(0)+4v(0.5)+2v(1.0)+\cdots+4v(4.5)+v(5.0)]$

(plug in the numbers from the table)

$x \approx (.5)(\frac{1}{3})[0+4(4.67)+2(7.34)+\cdots] \approx \boxed{44.735}$

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42) $\int_0^{20} \cos(\pi x)\,dx \qquad n = 10 \rightarrow \Delta x = 2$

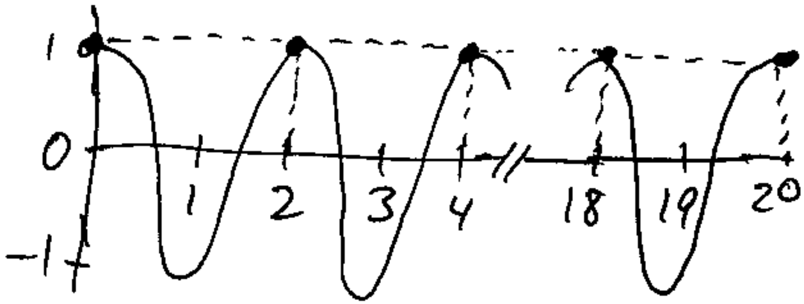
Trapezoidal:

$$\int \approx \cancel{2} \cdot \frac{1}{\cancel{2}}\left(\overset{f(0)}{\cos 0} + 2\overset{f(2)}{\cos 2\pi} + 2\overset{f(4)}{\cos 4\pi} + \cdots + \overset{f(20)}{\cos 20\pi}\right)$$

$$= (1+2+2+2+2+2+2+2+2+2+1) = \boxed{20}$$

Actual:

$$\int_0^{20} \cos(\pi x)\,dx = \frac{1}{\pi}\sin(\pi x)\Big|_0^{20} = \frac{1}{\pi}(\sin 20\pi - \sin 0) = \boxed{0}$$



The discrepancy is because the Trapezoidal Rule in this case (with $\Delta x = 2$) always selects the points that happen to be the peaks at $f(x) = 1$, even though the function spends half of the time below the x-axis! In order to get a more accurate result, n needs to be large enough so that Δx is small enough that it can take into account all of the important features of the function.