7.5: (2), 8, (14), 16, (17), 32, 40, (43), 44, 46, 48, (57).

2)

$$\int \frac{1+\cos x}{\sin x}\,dx$$

$$= \int (\csc x + \cot x)\,dx$$

$$= \ln|\csc x - \cot x| + \ln|\sin x| + C = \ln|1-\cos x| + C$$

8)

$$\int \frac{x}{\sqrt{3-x^4}}\,dx \qquad \text{let } u = x^2 \Rightarrow du = 2x\,dx$$

$$= \frac{1}{2}\int \frac{du}{\sqrt{3-u^2}}$$

$$= \frac{1}{2}\left(\sin^{-1}\left(\frac{u}{\sqrt{3}}\right)\right) + C$$

$$= \frac{1}{2}\cdot\sin^{-1}(x^2/\sqrt{3}) + C$$

14)

$$\int \frac{x}{x^4+x^2+1}\,dx$$

$$= \int \frac{x}{(x^2+x+1)(x^2-x+1)}\,dx$$

$$\frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1} = \frac{x}{(x^2+x+1)(x^2-x+1)}$$

$$\Rightarrow (Ax+B)(x^2-x+1) + (Cx+D)(x^2+x+1) = x$$

$$\left.\begin{aligned} x=0:&\ B+D=0\\ x=1:&\ A+B+3C+3D=1\\ x=-1:&\ -3A+3B-C+D=-1\\ x=2:&\ 6A+3B+14C+7D=2 \end{aligned}\right\}\quad \begin{aligned} &A=C=0\\ &B=-\tfrac{1}{2},\ D=\tfrac{1}{2} \end{aligned}$$

$$= \int\left(\frac{-1/2}{x^2+x+1} + \frac{1/2}{x^2-x+1}\right)dx$$

$$= -\frac{1}{2}\int\frac{dx}{x^2+x+1} + \frac{1}{2}\int\frac{dx}{x^2-x+1}$$

$$= -\frac{1}{2}\int\frac{dx}{(x+\frac{1}{2})^2+3/4} + \frac{1}{2}\int\frac{dx}{(x-\frac{1}{2})^2+3/4}$$

$$-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right) + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{x-1/2}{\sqrt{3}/2}\right) + C$$

$$-\frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

OR: let $u = x^2 \Rightarrow du = 2x\,dx$.

$\frac{1}{2}\int\frac{du}{u^2+u+1}$ and proceed with $\tan^{-1}$

$$= \frac{1}{2}\int\frac{du}{(u+\frac{1}{2})^2+3/4}$$

$$= \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{u+1/2}{\sqrt{3}/2}\right) + C = \frac{1}{\sqrt{3}}\cdot\tan^{-1}$$

16)

1) $\int e^{(x^{1/3})}\,dx$ let $a = x^{1/3} \Rightarrow x = a^3 \Rightarrow dx = 3a^2 da$.

$\int 3a^2 \cdot e^a\, da$. let $u = a^2 \Rightarrow du = 2a\,da$

$dv = e^a da \Rightarrow v = e^a$.

$= 3\left[a^2 \cdot e^a - 2\int a e^a\, da\right]$

$= 3\left[a^2 \cdot e^a - 2\left[a \cdot e^a - \int e^a da\right]\right]$

$= 3\left(a^2 \cdot e^a - 2(a \cdot e^a - e^a)\right) + C$

$= 3a^2 \cdot e^a - 6a \cdot e^a + 6e^a. + C$

$= 3x^{2/3} \cdot e^{x^{1/3}} - 6x^{1/3} \cdot e^{x^{1/3}} + 6e^{x^{1/3}} + C$.

17)

a) $\int \ln(1+x^2)\,dx$ let $u = \ln(1+x^2) \Rightarrow du = 2x/1+x^2 \cdot dx$

$x \cdot \ln(1+x^2) - \int 2x^2/1+x^2 \cdot dx$ $dv = dx \Rightarrow v = x$

$x \cdot \ln(1+x^2) - \int \left(2 - \frac{2}{1+x^2}\right)dx$.

$= x \cdot \ln(1+x^2) - 2x + 2 \cdot \tan^{-1}(x) + C$.

32)

a) $\int \frac{1}{x^3 - 8}\,dx$

$= \int \frac{1}{(x-2)(x^2+2x+4)}\,dx$

$= \int \left(\frac{1/12}{x-2} + \frac{-1/12 \cdot x - 1/3}{x^2+2x+4}\right)dx$

$\frac{1}{12}\ln|x-2| - \frac{1}{12}\int \frac{x+1}{x^2+2x+4}dx - \frac{1}{4}\int \frac{1}{(x+1)^2+3}\,dx$

$\frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4} = \frac{1}{(x-2)(x^2+2x+4)}$

$\Rightarrow A(x^2+2x+4) + (Bx+C)(x-2) = 1$

$x = 2: \; 12A = 1 \Rightarrow A = 1/12$

$x = 0: \; 4A - 2C = 1 \Rightarrow C = -1/3$.

$x = 1: \; 7A + B - C = 1 \Rightarrow B = -1/12$.

$\frac{1}{12}\ln|x-2| - \frac{1}{24}\ln|x^2+2x+4| - \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + C$.

'40)

2.) $\int \frac{1}{\sqrt{4y^2-4y-3}}\,dy$

$= \int \frac{1}{\sqrt{(2y-1)^2-4}}\,dy$ let $u = 2y-1 \Rightarrow du = 2\,dy$.

$= \int \frac{1/2}{\sqrt{u^2-4}}\cdot du$

$= 1/2 \cdot \ln\left|u + \sqrt{u^2-4}\right| + C$

$= 1/2 \cdot \ln\left|2y-1 + \sqrt{(2y-1)^2-4}\right| + C.$

43)

3)) $\int x^5 e^{-x^3}\,dx$ let $u = x^3 \Rightarrow du = 3x^2\,dx$

$dv = x^2 \cdot e^{-x^3} \Rightarrow v = -1/3 \cdot e^{-x^3}$

$= -\frac{x^3}{3}\cdot e^{-x^3} + \int x^2 \cdot e^{-x^3}\,dx$

$= -\frac{x^3}{3} e^{-x^3} - \frac{1}{3} e^{-x^3} + C.$

44)

4) $\int \frac{1+e^x}{1-e^x}\,dx$

$= \int \left(1 + \frac{2e^x}{1-e^x}\right) dx$

$= x + 2\ln(1-e^x) + C$

46)

1.) $\int \frac{x}{x^4 - a^4}\,dx$

OR: let $u = x^2 \Rightarrow du = 2x\,dx$. $\frac{1}{2}\int \frac{du}{u^2-a^4}$ ....

$= \int \frac{x}{(x^2-a^2)(x^2+a^2)}\,dx$

$\frac{A}{x-a} + \frac{B}{x+a} + \frac{Cx+D}{x^2+a^2} = \frac{x}{(x-a)(x+a)(x^2+a^2)}$

$= \int \frac{x}{(x-a)(x+a)(x^2+a^2)}\,dx$

$\Rightarrow A(x+a)(x^2+a^2) + B(x-a)(x^2+a^2) + (Cx+D)(x-a)(x+a) = x$

if $x = a$: $4a^3 \cdot A = a \Rightarrow A = 1/4a^2$.

if $x = -a$: $-4a^3 \cdot B = -a \Rightarrow B = 1/4a^2$

if $x = 0$: $a^3 \cdot A - a^3 B + D(-a^2) = 0 \Rightarrow D = 0.$

$C = -1/2a^2$ (by comparing x^3 coeff.)

(cont'd)

$$= \int \left(\left(\frac{1/4a^2}{x-a} \right) + \left(\frac{1/4a^2}{x+a} \right) + \left(\frac{-1/2a^2 \cdot x}{x^2+a^2} \right) \right) dx$$

$$= 1/4a^2 \cdot \ln|x-a| + 1/4a^2 \cdot \ln|x+a| - \frac{1}{4a^2} \cdot \ln|x^2+a^2| + C$$

48)

i.) $\int x^2 \tan^{-1} x \cdot dx$ let $u = \tan^{-1} x \cdot \Rightarrow du = \frac{1}{1+x^2} dx$

let $dv = x^2 dx \Rightarrow v = \frac{1}{3} x^3$.

$$\frac{1}{3} x^3 \cdot \tan^{-1} x - \int \frac{1}{3} \cdot \frac{x^3}{1+x^2} dx$$

$$\frac{1}{3} x^3 \cdot \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{1+x^2} \right) dx$$

$$\frac{1}{3} x^3 \cdot \tan^{-1} x - \frac{1}{3} \left(\frac{1}{2} \cdot x^2 - \frac{1}{2} \ln |1+x^2| \right) + C$$

$$\frac{1}{3} x^3 \cdot \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \ln |1+x^2| + C.$$

57)

i.) $\int x \cdot \sqrt[3]{x+c} \cdot dx$

$$= \int \left((x+c) \cdot \sqrt[3]{x+c} - c \cdot \sqrt[3]{x+c} \right) dx$$

$$= \int \left((x+c)^{4/3} - c \cdot (x+c)^{1/3} \right) dx$$

$$= \frac{3}{7} (x+c)^{7/3} - \frac{3c}{4} \cdot (x+c)^{4/3} + A.$$